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FRACTIONAL-ORDER MATHEMATICAL MODELING OF CLIMATE-DRIVEN VECTOR-BORNE DISEASE DYNAMICS IN PAKISTAN

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Abstract
Climate change is altering the environmental conditions that influence vector-borne disease transmission, creating major public-health challenges in climate-sensitive settings such as Pakistan. This study proposes a fractional-order mathematical model for climate-driven dengue transmission by integrating human–vector dynamics, climatic variability, and temporal memory. Temperature, rainfall, and relative humidity are incorporated into biologically meaningful transmission and vector parameters, while a Caputo fractional derivative represents nonlocal temporal dependence. The framework includes model calibration, equilibrium and stability analysis, the basic reproduction number (R_0), sensitivity and uncertainty analysis, validation, comparison with an equivalent integer-order model, and climate and intervention scenario simulations. The approach is intended to identify climate-sensitive transmission mechanisms and assess whether fractional dynamics improve empirical representation and prediction. The framework may support climate-informed surveillance, early-warning systems, and targeted vector-control planning in Pakistan. Because the supplied manuscript does not contain finalized empirical estimates or model outputs, numerical findings and hypothesis decisions remain placeholders and should be completed only after the dataset and simulations are finalized.

1. INTRODUCTION

Vector-borne diseases are shaped by interactions among pathogens, humans, vectors, environmental conditions, and socioeconomic factors. Dengue and malaria are particularly climate-sensitive because temperature, precipitation, humidity, and extreme weather can affect vector survival, reproduction, biting, pathogen development, and transmission. These relationships are nonlinear and geographically heterogeneous, and their effects are mediated by urbanization, land use, mobility, infrastructure, and public-health capacity (Klepac et al., 2024; Ryan et al., 2019; Semenza & Paz, 2021).

Pakistan is vulnerable to climate-related health risks because of climatic diversity, heatwaves, floods, changing precipitation, rapid urbanization, population density, and uneven public-health capacity. Dengue is especially relevant because Aedes aegypti and Aedes albopictus respond strongly to environmental conditions. Temperature can alter mosquito development and viral incubation, while rainfall and humidity influence breeding habitats and survival. Effects may also be delayed and nonlinear.

Pakistan-specific evidence indicates that climate change can alter the spatial and temporal distribution of dengue risk (Saeed et al., 2023). However, much existing work relies on statistical associations, climate suitability, or conventional integer-order models. Such approaches may not adequately represent the sequential and memory-dependent nature of vector development, pathogen incubation, infection, recovery, and intervention effects.

Fractional calculus provides a mechanism for representing temporal memory and hereditary effects. A fractional order $0 < \alpha \leq 1$ allows the model to retain the classical formulation when $\alpha = 1$ while introducing nonlocal temporal dependence when $0 < \alpha < 1$. Recent dengue studies support the use of fractional-order models for transmission, stability, sensitivity, and intervention analysis (Meetei et al., 2024; Usman et al., 2024). Integrating climate dependence with fractional dynamics therefore offers a potentially useful framework for Pakistan.

2. Problem Statement

Pakistan faces increasing climate-sensitive dengue risk, yet translating climatic information into quantitative transmission predictions remains difficult. Existing climate–disease studies commonly use regression, time-series analysis, ecological modelling, or conventional compartmental systems. These approaches provide important evidence but may not explicitly represent biological transmission mechanisms, delayed effects, or historical dependence.

A key methodological gap exists at the intersection of climate science, epidemiology, and fractional calculus. Pakistan-specific climate–dengue studies have established changing spatial and temporal risk, while fractional dengue studies have demonstrated the usefulness of memory-dependent dynamics. However, limited work combines Pakistan-specific climate drivers, a mechanistic human–vector system, fractional memory, empirical calibration, model comparison, and intervention scenarios within one framework. This study addresses that gap.

3. Research Questions

- How do temperature, precipitation, humidity, and other climatic variables influence vector-borne disease transmission in Pakistan?
- Does a fractional-order model represent climate-driven disease dynamics more effectively than an equivalent integer-order model?
- How does the fractional order affect temporal persistence, stability, and epidemic behaviour?
- Which climatic and epidemiological parameters most strongly influence R_0 and disease prevalence?
- How do projected climatic variations affect the intensity, timing, and persistence of outbreaks?
- Which vector-control and public-health interventions are most effective under alternative climate scenarios?

4. Research Objectives

General Objective

To develop and analyze a fractional-order mathematical model for describing and predicting climate-driven vector-borne disease dynamics in Pakistan.

Specific Objectives

- Formulate a climate-dependent fractional-order compartmental model for dengue transmission.
- Incorporate temperature, precipitation, and humidity into biologically meaningful parameters.
- Establish positivity, boundedness, existence, uniqueness, and stability properties.
- Derive R_0 and identify conditions associated with disease persistence or elimination.
- Estimate parameters using appropriate Pakistani epidemiological and climate data and assess model fit.
- Quantify sensitivity to climatic, epidemiological, and intervention parameters.
- Compare the fractional-order model with an equivalent integer-order model.
- Simulate alternative climate and intervention scenarios.

- Identify climate-sensitive parameters and interventions relevant to surveillance and control.

5. Significance of the Study

The study contributes theoretically by integrating fractional calculus with climate-driven vector-borne disease transmission and by linking environmental forcing to epidemiological mechanisms. Methodologically, it combines climate-sensitive modelling and fractional-order epidemiology within a Pakistan-specific framework and directly compares fractional and integer formulations. From a public-health perspective, the framework may support earlier surveillance, targeted vector control, environmental management, and climate-informed preparedness. At the policy level, it supports integration of meteorological and epidemiological information and the development of climate-resilient health planning.

6. Literature Review

6.1 Climate and Vector-Borne Disease Dynamics

Climate affects vector survival, development, reproduction, biting, distribution, and pathogen development. Temperature effects are often nonlinear: moderate increases can enhance transmission within a favourable biological range, whereas excessive heat can reduce vector survival. Rainfall can increase breeding-site availability but excessive precipitation may eliminate immature habitats. Humidity can increase mosquito longevity. Climate effects also interact with land use, urbanization, mobility, behaviour, and infrastructure (de Souza & Weaver, 2024; Kirk et al., 2024).

6.2 Dengue Modelling and Pakistan

Compartmental dengue models commonly represent susceptible, exposed, infectious, and recovered humans and susceptible, exposed, and infectious vectors. These models support analysis of equilibria, R_0, stability, and interventions, but classical integer-order formulations generally assume memoryless instantaneous dynamics. Saeed et al. (2023) demonstrated that climate change can modify dengue risk across Pakistan, providing an empirical basis for climate-sensitive modelling. The present study extends this direction by emphasizing dynamic transmission and temporal memory.

6.3 Fractional-Order Epidemiological Modelling

Fractional calculus generalizes differentiation to non-integer orders and can represent nonlocal temporal dependence. In epidemiology, fractional models have been used to study disease thresholds, stability, sensitivity, calibration, and intervention. For a Caputo derivative, $\alpha = 1$ gives the classical model, whereas $0 < \alpha < 1$ introduces memory. Recent dengue studies demonstrate the feasibility of fractional-order transmission and intervention modelling (Meetei et al., 2024; Usman et al., 2024).

6.4 Climate-Dependent Fractional Models

Recent work has integrated climatic variables into fractional-order infectious-disease models, including temperature and rainfall. Such studies support the feasibility of combining environmental forcing and memory within one mathematical system. However, dengue differs biologically from malaria and requires an Aedes-specific human–vector formulation. The proposed study therefore adapts the general climate–fractional approach to Pakistan-specific dengue dynamics rather than directly reproducing another disease model.

6.5 Research Gap

- Climate–fractional integration gap: climate-driven and fractional epidemic models are often studied separately.
- Pakistan-specific modelling gap: limited fractional mechanistic models are calibrated to Pakistani dengue and climate data.
- Memory gap: conventional climate-driven models generally do not explicitly represent hereditary temporal effects.
- Decision-support gap: climate forcing, memory, sensitivity, validation, and intervention scenarios are rarely integrated in one Pakistan-focused framework.

7. Hypotheses

- H1: Favourable temperature, precipitation, and humidity conditions are positively associated with vector abundance and disease transmission.
- H2: The climate-driven fractional-order model demonstrates better empirical fit and predictive performance than the equivalent integer-order model.
- H3: A fractional order below one significantly alters disease persistence and temporal dynamics compared with $\alpha = 1$.
- H4: Temperature-sensitive vector and pathogen parameters significantly influence R_0 under favourable climatic conditions.
- H5: Increased vector-control effectiveness reduces R_0 and predicted disease prevalence.
- H6: Combined climate-informed surveillance and vector control produce greater reductions in predicted transmission than either strategy alone.
- H7: Climate scenarios with greater suitability for vectors increase the predicted intensity and/or duration of transmission.
- H8: Sensitivity to climatic parameters varies across geographical and seasonal conditions in Pakistan.

8. Methodology

8.1 Research Design and Setting

The study uses a quantitative, retrospective, longitudinal mathematical-modelling design. Dengue is the principal disease considered, with monthly epidemiological and climatic observations from Pakistan. Where data completeness permits, major affected urban and peri-urban areas across Punjab, Sindh, Khyber Pakhtunkhwa, and other administrative regions are incorporated. A national model is followed by geographical stratification where sufficient observations are available.

8.2 Data Sources and Integration

The study uses secondary longitudinal data. Epidemiological variables include monthly dengue cases, incidence, deaths where available, location, reporting period, and population estimates. Climate variables include mean, minimum, and maximum temperature, rainfall, relative humidity, and available extreme-weather indicators. Climate variables are temporally aligned with disease observations and lagged variables are constructed to capture delayed environmental effects. Data are screened for duplicates, missing values, inconsistent geographic identifiers, implausible observations, and measurement inconsistencies.

8.3 Sampling and Analytical Dataset

Purposive data selection is used to identify geographical units and periods satisfying predefined data-availability criteria. All eligible observations within the defined study period are included rather than selecting a random subsample. The final sample size should be reported only after the study period and eligible locations are finalized. The source manuscript currently contains [N] placeholders and therefore does not support reporting a fabricated sample size. A chronological 70% calibration, 15% validation, and 15% testing split is proposed.

8.4 Model Specification

Humans are divided into susceptible (S_h), exposed (E_h), infectious (I_h), and recovered (R_h) compartments; vectors are divided into susceptible (S_v), exposed (E_v), and infectious (I_v). The system uses a Caputo derivative of order $0 < \alpha \leq 1$. Climate-dependent functions modify vector recruitment, mortality, biting, and pathogen-development parameters.

The general structure is:

${}^{c}D^{\alpha}S_h = \Lambda_h - \lambda_h(t)S_h - \mu_h S_h$; ${}^{c}D^{\alpha}E_h = \lambda_h(t)S_h - (\sigma_h+\mu_h)E_h$; ${}^{c}D^{\alpha}I_h = \sigma_h E_h - (\gamma_h+\mu_h+\delta_h)I_h$; ${}^{c}D^{\alpha}R_h = \gamma_h I_h - \mu_h R_h$.

For vectors: ${}^{c}D^{\alpha}S_v = \Lambda_v(t) - \lambda_v(t)S_v - \mu_v(t)S_v$; ${}^{c}D^{\alpha}E_v = \lambda_v(t)S_v - (\sigma_v+\mu_v(t))E_v$; ${}^{c}D^{\alpha}I_v = \sigma_v E_v - \mu_v(t)I_v$.

The forces of infection are represented generally by $\lambda_h(t)=a(t)b_{vh}(t)I_v/N_h$ and $\lambda_v(t)=a(t)b_{hv}(t)I_h/N_h$. Climate-response functions allow $a(t)$, $\mu_v(t)$, and $\sigma_v(t)$ to depend on temperature, rainfall, and humidity.

8.5 Estimation and Numerical Solution

Unknown parameters are estimated using epidemiological and climate observations through nonlinear least squares, maximum likelihood, or Bayesian estimation as appropriate. Parameters are constrained to biologically meaningful ranges. The fractional system is solved numerically using an appropriate method such as the fractional Adams–Bashforth–Moulton predictor–corrector scheme. Numerical convergence is assessed using progressively smaller step sizes.

8.6 Data Analysis and Validation

Analysis includes descriptive statistics, seasonal and temporal analysis, lagged climate–disease associations, model calibration, R_0 derivation, equilibrium and stability analysis, global sensitivity analysis using Latin Hypercube Sampling/PRCC where feasible, and comparison with the integer-order model. Performance is evaluated using RMSE, MAE, MAPE, R^2, AIC, BIC, and out-of-sample predictive accuracy. Climate scenarios include baseline, warming, altered rainfall, and combined climate conditions; intervention scenarios vary vector mortality/recruitment and climate-informed surveillance.

8.7 Reliability, Validity, and Ethics

Reliability is strengthened through data-quality checks, repeated estimation, convergence testing, sensitivity analysis, and replication. Validity is assessed through biological correspondence of model compartments, consistency with established literature, comparison with observed incidence, and out-of-sample testing. The study uses aggregated secondary data and does not involve direct recruitment or personally identifiable patient information. Restricted individual-level data, if later used, require appropriate ethical approval and authorization.

9. Data Analysis and Interpretation

The supplied manuscript does not contain finalized empirical results. Accordingly, this shortened version retains the planned analysis without fabricating numerical findings.

9.1 Descriptive and Temporal Analysis

Dengue incidence and temperature, rainfall, and humidity are summarized using means, standard deviations, ranges, and coefficients of variation. Monthly patterns are examined to identify seasonal peaks and to determine whether climatic conditions precede changes in incidence.

9.2 Climate–Disease Associations

Pearson or Spearman correlations and lagged analyses are used to assess relationships between climatic variables and dengue incidence. Lagged effects are interpreted cautiously because correlation does not establish causality and climatic variables may be correlated.

9.3 Model Calibration, R_0, and Sensitivity

The fractional model is fitted to observed incidence and its parameters are estimated with uncertainty intervals where appropriate. R_0 is derived using the next-generation matrix approach; $R_0 < 1$ indicates declining transmission under the specified conditions, whereas $R_0 > 1$ indicates potential persistence. Sensitivity analysis identifies the parameters most influential for R_0 and predicted incidence.

9.4 Fractional versus Integer-Order Performance

The fractional model is compared with the equivalent integer-order model. A claim of superiority should be made only if lower prediction errors and/or improved information criteria are demonstrated, particularly on independent test data. Calibration-only improvement may indicate overfitting.

9.5 Climate and Intervention Scenarios

The calibrated model is used to examine alternative temperature and rainfall conditions and different levels of vector-control effectiveness. Because biological responses may be nonlinear, scenario results should be interpreted as model-based projections rather than deterministic forecasts.

Important placeholders in the source manuscript—such as [N], α, R_0, RMSE, MAE, R^2, sensitivity indices, and scenario outcomes—must be replaced with actual outputs after the dataset and simulations are finalized. filecite turn1file0 L364-L374

10. Discussion

The study is designed to determine whether climatic variability and temporal memory jointly explain dengue dynamics in Pakistan. Climate effects are expected to operate through vector development, survival, abundance, biting, and pathogen development, with possible lags between environmental exposure and reported human infection. These mechanisms support nonlinear and lagged climate-response functions rather than simple linear associations.

The fractional-order component provides a mechanism for representing historical dependence. If $\alpha < 1$ and the fractional model demonstrates superior out-of-sample performance, the findings would provide empirical support for measurable memory in the disease system. If predictive improvement is negligible, the additional complexity of the fractional formulation should not be considered automatically justified.

Climate-sensitive R_0 should be interpreted as varying with environmental conditions rather than as a fixed national value. Sensitivity analysis can identify influential parameters such as temperature, vector recruitment, biting rate, vector mortality, and pathogen development, thereby informing intervention priorities. The interpretation should remain cautious because climate does not act independently of urbanization, mobility, water storage, sanitation, behaviour, and surveillance intensity.

The source manuscript appropriately emphasizes that final discussion claims must be based on actual estimates and validation statistics rather than theoretical expectations; numerical results should not be invented. filecite turn1file0 L357-L374

11. Conclusion

This study develops a climate-driven fractional-order mathematical framework for dengue transmission in Pakistan by integrating climatic variability, human–vector dynamics, and temporal memory. The framework is designed to estimate climate-sensitive parameters, derive R_0, evaluate stability and sensitivity, compare fractional and integer formulations, validate predictions, and simulate climate and intervention scenarios.

The principal contribution is the integration of climate forcing, vector transmission, and fractional temporal memory within a Pakistan-specific modelling framework. The approach may provide a quantitative basis for climate-informed surveillance, early-warning systems, and targeted vector-control planning. Final empirical conclusions, however, must be based on the completed dataset, estimated parameters, model-performance statistics, and scenario outputs.

12. Implications

Theoretical implications include extending conventional compartmental epidemiology by integrating environmental forcing and temporal memory. Practical implications include using climatic indicators to anticipate transmission risk, prioritizing high-risk districts, strengthening pre-epidemic surveillance, improving larval-source management, and preparing health services before expected peaks. Policy implications include stronger integration of meteorological and epidemiological databases and geographically disaggregated surveillance.

13. Recommendations

- Develop climate-informed dengue early-warning systems integrating temperature, rainfall, humidity, and surveillance data.
- Prioritize vector-control activities according to parameters identified as highly sensitive.
- Strengthen pre-epidemic and longitudinal vector surveillance.
- Integrate meteorological, epidemiological, demographic, and entomological databases.
- Compare fractional and conventional models empirically rather than assuming fractional superiority.
- Adopt geographically targeted intervention planning.
- Strengthen community environmental management, including standing-water and waste management.
- Regularly recalibrate models as new climate, epidemiological, and entomological data become available.

- Integrate vector-borne disease risks into climate-adaptation and health policy.

14. Limitations

- Secondary surveillance data may underestimate true dengue incidence because of asymptomatic infections, diagnostic limitations, and healthcare-seeking differences.
- Routine mosquito-abundance data are limited, so some vector parameters may be inferred rather than directly observed.
- Climate variables do not fully capture urbanization, land use, water storage, sanitation, mobility, housing, or behaviour.
- The basic model simplifies dengue biology, including serotypes, immunity, antibody-dependent enhancement, and heterogeneous vector competence.
- Parameter identifiability and uncertainty may affect model projections.
- Climate–disease relationships can vary geographically, limiting the generalizability of national estimates.
- Future climate projections contain substantial uncertainty.
- Fractional models add mathematical and computational complexity.

15. Future Research

Future studies should incorporate high-resolution entomological observations; spatially explicit fractional reaction–diffusion, metapopulation, or network models; multi-serotype dengue dynamics; climate-model ensembles; hybrid fractional–machine-learning approaches; Bayesian parameter estimation; comparison of alternative fractional operators; and real-time district-level early-warning applications.

16. Overall Contribution

The study moves beyond asking whether climate is associated with dengue and instead examines how climatic variability interacts with vector ecology and temporal memory to shape transmission intensity, persistence, and predictability. Its principal methodological contribution is a unified Pakistan-focused framework that can empirically test whether memory-dependent fractional dynamics improve the representation of climate-sensitive dengue transmission.

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