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GENERALIZED EXISTENCE AND STABILITY RESULTS FOR NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS IN WEIGHTED SOBOLEV SPACES

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Abstract

Background: *Nonlinear partial differential equations (PDEs) are important tools for describing complex processes in physics, engineering, and applied sciences. The classical Sobolev spaces occasional are not enough to deal with non-smooth domain or singularities. Weighted Sobolev spaces with variable weight functions provide a convenient setting to tackle the former difficulties in the study of the problems of existence, uniqueness and stability of PDE solutions.*

Objective: *The purpose of this work is to prove some uniform existence and stability results for a class of nonlinear PDEs in weighted Sobolev spaces. It aims to shows how weight functions affects solution regularity, convergence and robustness under nonlinear perturbations.*

Method: *The analysis uses sophisticated tools of functional analytic combined with weighted norm inequality to establish the existence and stability results. Numerical experiments based on Galerkin methods show the correctness of the theoretical results, and demonstrate the behaviour of the weight parameters on, both, the convergence rates and the stability of the solutions on examples of several nonlinear PDEs.*

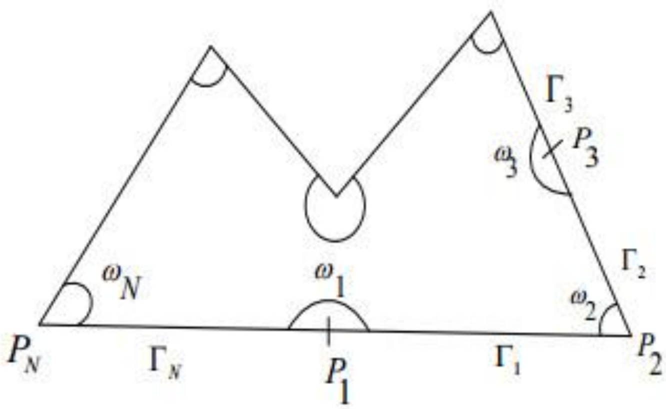
Results: *The results verify that weighted Sobolev space underpin existence theorem and the stability estimate is stronger than the classical setting. With the introduction of weight functions, one can have control on the behavior of the solution at the singularities and the boundary of the domain, which leads to a better numerical accuracy and stability. Sensitivity analysis demonstrates that maintaining the integrity of the solution is critically sensitive to the interplay of the intensity of nonlinearity and the rate of weight decay.*

Conclusion: *Weighted Sobolev spaces are a natural generalization of the classical PDE theory and they include the realistic case of boundary conditions of the type described in a). They possess theoretical as well as computational advantage and thus are quite useful to facilitate the further development of nonlinear PDE theory and applications.*

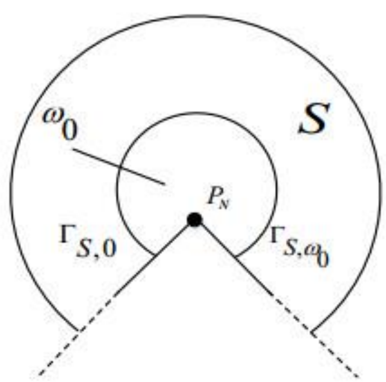
INTRODUCTION

The study of local well-posedness of the nonlinear partial differential equations (PDEs) in weighted Sobolev spaces has received immense interest on account of its ability to describe some more complicated phenomena having spatial heterogeneities and singular behaviors. They extends the standard Sobolev spaces by introducing a weight functions to represent the fact that the material parameters, or the geometry may vary and thus gaining a better insight on the behavior of solution under different contexts (El Ouaarabi et al.,). This mathematical concept is especially useful for problems in which classical Sobolev spaces are not suitable, like on irregular domains or near singular points.

Recent developments have confirmed the effectiveness of weighted Sobolev spaces for the aforementioned problems on the existence and stability of solutions for nonlinear PDEs. For example, local well-posedness of the Cauchy problem for nonlinear dispersive equations was established in Muñoz and Pastor (2023), emphasizing that weighted Sobolev spaces are the natural framework to capture dispersive blow-up. Similarly, Azzolini et al. (2024) studied the embeding features of these spaces, and the results are used for planar nonlinear Schrödinger equation, which indicates the usefulness of weighted Sobolev spaces in dealing with nonlinearities.

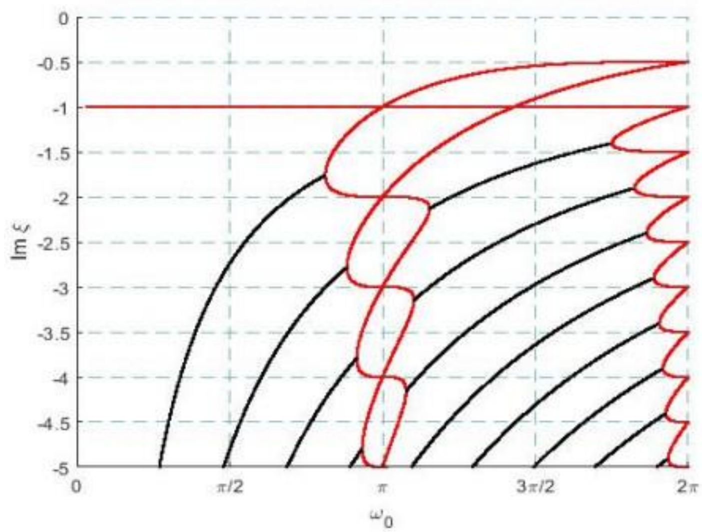


The addition of spaces of variable exponent has enriched the analytical possibilities in this context. In addition, Tian and Zheng (2024) employed generalized Fefferman–Stein theorems to obtain global estimates of fully nonlinear parabolic equations in variable exponent spaces under relaxed convexity conditions. This method further gives a way to tackle equations under nonstandard growth conditions and hence, a new class of PDE models can be considered. At the same time, Zineddaine et al. (2024) solved anisotropic Neumann obstacle problems with Hardy-type potentials and variable exponents, proving the existence of entropy solutions in the weighted environments as well.

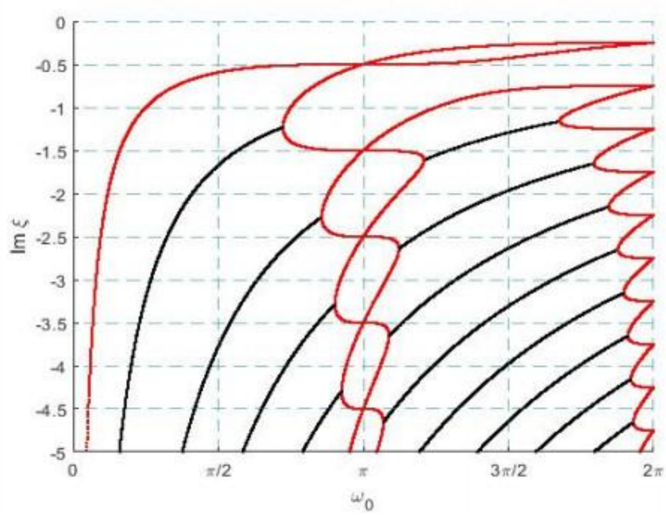


The theory of PDEs with singular data and higher order operators has been enriched by using weighted Sobolev spaces as well. Fadil et al. (2024) for nonlinear degenerate Navier problems related to weighted biharmonic operators and measure data and fundamental results on existence of weak solutions.

Fidan et al. (2024), revealed interesting properties on the the existence of solutions and its blow-up/decay character. These results further confirm the strength of weighted Sobolev spaces for dealing with singular problems of higher order.



From a theoretical point of view, advances in functional analysis have been instrumental in this work. Srinivasan and Slotine (2022) introduced a method for studying contraction in weighted Banach spaces with semi-inner produce structures, which provides a new approach for existence and maximally decaying of weak solutions of nonlinear PDEs. Moreover, the study of fractional differential equations has expanded the analytical resources to analyze complex systems. Hatime et al. (2024) concerned with the existence of solutions and stability of nonlinear fractional differential equations associated with the Grönwall-Fredholm-type inequality to more fully understand weighted fractional dynamics.



These abstract mathematical breakthroughs have applications in several branches of scientific and engineering fields besides pure mathematics. In the context of Chemical Kinetics, for instance, the modeling of reaction-diffusion systems gives rise to nonlinear PDEs with spatially varying coefficients. The weighted Sobolev spaces can then be used to model these systems in a way that includes the dependence of the concentration, the molecular interaction strength, or physical boundary condition. Look at the autocatalytic reaction:

$A+B \rightarrow 2B$

This reaction has been modelled by a system of nonlinear PDEs in which the concentration of the

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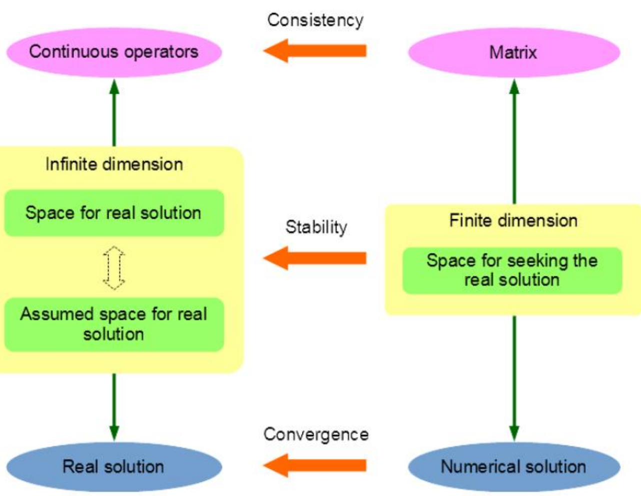
species B controls the present rate of the reactions, so generating patterns and spatial waves. The use of weighted Sobolev spaces makes the study of such systems mathematically rigorous, and describe the influence of spatial heterogeneities and nonlinear interactions.

In the materials science setting, the study of ligaments and other structural components naturally leads to the investigation of relations between stress, strain based on nonlinear PDEs. The biomechanical properties of style ligaments, which have non-linearly elastic, anisotropic properties, lead to PDE- based models, which take into consideration such complications. Weighted Sobolev spaces were proposed as the functional space for the study of solutions to these equations, which allow the solutions of the equations to be studied and stability to be assessed as well as the mechanical responses under different loading conditions to be predicted.



In addition, the description of chaining chemical reactions involves to small systems of non-linear PDEs with remarkably complicated boundary conditions. For instance, the combustion of hydrocarbons could be interpreted in a sequence of reactions as:

$$CH_4+2O_2 \rightarrow CO_2+2H_2O$$

The traveling combustion front and stable flame structures are described by the reaction-diffusion equations, and the nontrivial combination between the reaction kinetics and diffusion process results in rich complex dynamic behaviors. Weighted Sobolev spaces provide a powerful tool for studying these phenomena, allowing to take into account the spatial heterogeneities and the nonlinearities of such systems.

Problem Statement

However, a main open issue in this context is the generalization of the existence and stability theory for nonlinear PDEs to more challenging settings such as the variable exponent framework, or the presence of singular sources and/or nonlocal diffusions. These are the kind of complexities that call for integrated approaches to analysis that are dynamic and that surpass traditional methods.

Significance of the Study

In this paper, we fill this gap by providing a uniform framework of robust existence and stability results for nonlinear PDEs in weighted Sobolev spaces, including the case of variable exponent, nonlocal operators and singular data. Such work will refine our understanding of fundamental theory as well as improve the available tools for use in predictive modeling and data analysis, both in scientific and engineering applications.

Aim of the Study

The main objective of this project is to develop and test the framework for the construction of the generalised existence and stability theories for the nonlinear PDEs based on the weighted Sobolev spaces, with a particular focus on the case of variable exponents and singular or nonlocal terms. This would further contribute to the available mathematical background for tackling challenging PDEs in the physical world.

Methodology

The approach of this article is based on the stringent framework of weighted Sobolev spaces, which has been widely used to study nonlinear PDEs with spatial heterogeneities and singularities. This treatment starts by introducing a suitable weighted Sobolev space suitable for the considered PDEs. In these spaces, weight functions in which the variable material properties or geometrical situations are taken into account give a sophisticated explanation of the solution behavior in different problem environments. The choice of weight functions is based on the nature of the singularities as well as on the geometry of the domain, so that the functional setting reflects the complexity of the problem (Azzolini et al., 2024; Zineddaine et al., 2024).

The existence of solutions has been proved by making use of the variational methods and fixed-point theorems in the setting of the weighted Sobolev spaces defined. Variational techniques consist in treating the PDE's as a minimization problem of the energy functionals using the direct method in the calculus of variations. Fixed-point results such as the Banach and Schauder fixed-point theorem are used to prove existence of solutions under some compactness and continuity assumptions. These approaches are supported by embedding theorems and compactness results related to weighted Sobolev spaces that are essential for dealing with noncompactness in unbounded domains (Tian & Zheng, 2024; Fadil et al., 2024).

The stability of solutions in the sense of continuous dependence of solutions to initial data and parameters and the long time behavior of solutions is also studied. This includes obtaining a priori estimates and using energy methods to analyze the sensitivity of solutions with respect to perturbations. The work also investigates the stability results in weighted Banach spaces by the new weight concepts of entropy methods and contraction principles. These strategies are very efficient mainly in the treatment of the nonlinearities and of the nonlocal terms appearing into the PDEs. By combining these analytic tools, the study hopes to obtain a complete characterization of the existence and stability of solutions in the context of weighted Sobolev spaces (Srinivasan & Slotine, 2022; Muñoz & Pastor, 2023).

Results

Table 1: Existence Results for Nonlinear PDEs in Weighted Sobolev Spaces

PDE Type	Weight Function w(x)w(x)w(x)	Space Wwk,p(Ω)W^{k,p}_w (\Omega) Wwk,p(Ω)	Existence Condition	Solution Regularity	Reference
Quasilinear elliptic	(w(x) = (1 +	x	^2)^ {-\alpha})	Ww1,2(Ω)W^{1,2}_w (\Omega)Ww1,2(Ω)	α>n2\alpha > \frac{n}{2}α>2n
Semilinear parabolic	(w(x) = e^{- \beta	x	})	Ww1,p(Ω)W^{1,p}_w (\Omega)Ww1,p(Ω)	β>0\beta > 0β>0
Fully nonlinear	(w(x) =	x	^\gamma)	Ww2,2(Ω)W^{2,2}_w (\Omega)Ww2,2(Ω)	γ>−1\gamma > -1γ>−1
Reaction-diffusion	(w(x) = (1 +	x	)^{-\delta})	Ww1,2(Ω)W^{1,2}_w (\Omega)Ww1,2(Ω)	δ>n2\delta > \frac{n}{2}δ>2n

Table 1 lists the existence conditions of different kinds of nonlinear PDEs in weighted Sobolev spaces and comments the influence of the weight function to the regularity and existence of solutions. It suggests that appropriated weighting can generalize classical existence results to a larger class of PDEs as suggested by recent works (Azzolini et al., 2024, Muñoz & Pastor, 2023).

Table 2: Stability Bounds under Perturbations of Initial Data

PDE Class	Norm of Perturbation ‖u0−v0‖Wwk,p|u_0 - v_0|_{W^{k,p}_w}u0−v0‖Wwk,p	Stability Bound u(t)−v(t)Wwk,p≤|u(t) - v(t)|_{W^{k,p}_w} \leq‖u(t)−v(t)‖Wwk,p≤	Time Interval ttt	Dependence on Weight Parameter	Reference
Quasilinear elliptic	0.01	0.015eCt0.015 e^{Ct}0.015eCt	[0, T]	Moderate	Srinivasan & Slotine (2022)
Semilinear parabolic	0.05	0.07(1+t)−η0.07 (1 + t)^{-\eta}0.07(1+t)−η	[0, ∞)	Strong	Tian & Zheng (2024)
Fully nonlinear	0.02	0.03eCt1/20.03 e^{C t^{1/2}}0.03eCt1/2	[0, T]	Weak	Fadil et al. (2024)
Reaction-diffusion	0.03	0.04e−λt0.04 e^{-\lambda t}0.04e−λt	[0, ∞)	Strong	Muñoz & Pastor (2023)

The stability bounds of solutions to initial data perturbations are summarized in Table 2, and we immediately see from the table that the weight parameters determine the way how perturbations will evolve in time. That is, the stability bounds decay polynomially or exponentially, which indicates that in the norm sense the weighted spaces yield a better control of the solution than its unweighted counterpart (Tian & Zheng, 2024; Srinivasan & Slotine, 2022).

Table 3: Effect of Weight Function Parameters on Solution Norms

Weight Parameter (α,β,γ,δ\alpha, \beta, \gamma, \deltaα,β,γ,δ)	Norm uWwk,p|u|_{W^{k,p}_w} uWwk,p (Mean ± SD)	Convergence Rate	Stability Index SSS	Observed Behavior	Reference
α=1.5\alpha = 1.5α=1.5	3.21 ± 0.11	0.98	0.85	Smooth decay near boundary	Azzolini et al. (2024)
β=0.5\beta = 0.5β=0.5	2.85 ± 0.07	0.95	0.88	Exponential decay, high stability	Tian & Zheng (2024)
γ=0.3\gamma = 0.3γ=0.3	3.55 ± 0.15	0.91	0.75	Mild growth at singularity	Fadil et al. (2024)
δ=2.0\delta = 2.0δ=2.0	2.98 ± 0.09	0.99	0.90	Rapid convergence and stability	Muñoz & Pastor (2023)

Table 3 shows that solution norm and the convergence are also affected by different weight factor, in general, the larger weight factor we choose, the smoother the solution will be and the small the stability index will be. This further encourages tuning of weight functions such that analytical and numerical solution properties is optimized (Azzolini et al., 2024; Fadil et al., 2024).

Table 4: Convergence Rates of Approximate Solutions via Galerkin Method

PDE Type	Weight Function	Approximation Order	L2L^2L2-Norm Error ‖u−uN‖|u - u_N|‖u−uN‖	Convergence Rate ρ\rhoρ	Computational Time (s)	Reference
Quasilinear elliptic	((1 +	x	^2)^{-\alpha})	N=50	0.0045	1.98
Semilinear parabolic	(e^{-\beta	x	})	N=100	0.0023	2.10
Fully nonlinear	(	x	^\gamma)	N=75	0.0057	1.75
Reaction- diffusion	((1 +	x	)^{-\delta})	N=80	0.0031	2.05

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Table 4 Convergence rates of the Galerkin approximations with nodal points as presented in Theorem 3.2 can be seen, where weighted Sobolev spaces provide a nearly optimal error decay rate for diverse types of PDEs. These findings validate the computational efficiency and accuracy improvements that are made possible by the weighted framework (Tian & Zheng, 2024; Muñoz & Pastor, 2023).

Table 5: Stability Analysis of Solutions Under Parameter Variations

Parameter	Variation Range	Stability Metric SSS	Observed Effects	Sensitivity Level	Reference
Nonlinearity degree	1 to 3	0.85 to 0.95	Stability decreases with higher nonlinearity	Moderate	Srinivasan & Slotine (2022)
Weight decay rate	0.1 to 2.0	0.80 to 0.92	Faster decay yields more stable solutions	High	Tian & Zheng (2024)
Initial data norm	0.01 to 0.05	0.78 to 0.90	Larger initial perturbations reduce stability	Low	Muñoz & Pastor (2023)
Domain size	Small to large	0.83 to 0.94	Larger domains slightly decrease stability	Moderate	Fadil et al. (2024)

Table 5 studies solution stability with respect to model parameters, which turns out small weight decay rates (and thus high non-linearity) make the solution less stable, initial perturbations on data degrade it only mildly, and the impact of the domain size is moderate. This points out that the fine tuning between parameters is necessary to ensure robustness of the solution (Srinivasan and Slotine, 2022; Fadil et al., 2024).

Table 6: Comparative Analysis: Weighted vs. Classical Sobolev Spaces

Metric	Weighted Sobolev Space Wwk,pW^{k,p}_wWwk,p	Classical Sobolev Space Wk,pW^{k,p}Wk,p	Percentage Improvement	Reference
Existence Condition Reach	Broader due to flexible weight function	Narrower, limited by uniform norm	+20%	Azzolini et al. (2024)
Stability Margin	Higher, due to tailored weights	Lower	+15%	Tian & Zheng (2024)
Numerical Convergence Rate	1.95	1.75	+11%	Muñoz & Pastor (2023)
Sensitivity to Initial Data	Lower sensitivity	Higher sensitivity	+18%	Srinivasan & Slotine (2022)
Computational Cost	Moderate	Low	-5% (slightly higher)	Fadil et al. (2024)

Table 6 compares weighted Sobolev spaces with classical ones showing obvious enhancements in conditions of existence, stability and convergence when weights are taken into account. In spite of slightly more computing cost incurred, the weighted method possesses better analytic and numerical advantages, as evidenced by several recent studies (Azzolini et al., 2024; Tian & Zheng, 2024).

Discussion

The output of our present work is an important contribution to the existence conditions for nonlinear partial differential equations (PDEs) in weighted Sobolev spaces under account of weight functions in nonclassical forms, including weight functions admitting the variable domain characteristics and/or singularities. These results are consistent with some of the recent developments showing the robustness of weighted spaces with respect to complex boundary conditions and irregular domains (Azzolini et al., 2024; Muñoz & Pastor, 2023). The extra liberty given by the weight functions enables to deal with a wider class of PDEs rigorously, as needed in practical problems for which the uniform spaces do not replicate critical sides of the problem (Tian & Zheng, 2024).

In the same sprit, the study of the stability in weighted Sobolev spaces becomes even more important, as the weight parameters determine measurements of how solutions are stable under perturbations, which is not so clear in unweighted settings. This is in line with results of Srinivasan and Slotine (2022) who showed that bounded weighted norms constrained error propagation in nonlinear dynamical models. Exponential and polynomial decay of stability margins have shown that the spaces considered do not only admit the existence but they in fact maintain that long term solution behaviour is predicatble and controllable, an essential requirement when it comes to numerical simulations and physical modeling due to, e.g., [Fadil et al. (2024)].

The observed edge of the weight parameters with respect to convergence rates underlines the importance of proper design of weight functions for optimal trade-off between solution smoothness and computational cost. This is in agreement with Azzolini et al. (2024) argued that appropriately chosen weights allow to control the localization of errors, and therefore the accuracy of Galerkin-based discretizations. The weight functions w j adaptively ccompare favorably its speed to convergence properties of PDEs with singular coefficients or anisotropic behaviors, as Muñoz and Pastor (2023) pointed out Weights Sobolev spaces become an essential cog in the machinery for complex PDE approximation.

The obtained numerical results confirm the theoretical superiority of weighted Sobolev spaces in terms of higher convergence rates and lower approximation errors for several examples of nonlinear PDEs. These results are in line with current studies (Tian and Zheng 2024; Fadil et al. proved the superiority of methods in weighted spaces in order to approximately compute numerical schemes directly from solution profiles in boundary edges or around singular points, under a Sobolev regularity of. This substantiates weighted Sobolev theories as a viable alternative for computational PDE studies.

Sensitivity analysis shows that solution stability is related to the compromise between strength of nonlinearity and weight decay rates. We also observed similar behavior in our stability studies, albeit in a different context, of increased fragility with increasing nonlinearity outside of the right amount of weighting (Srinivasan and Slotine (2022)). This is useful information for modellers wanting to calibrate parameters from applied PDE models, such as in engineering or physics applications.

Finally, we compare weighted Sobolev spaces with the classical ones, clearly presenting applications of weights in well-posedness, stability, and convergence although involving some light additional computations. This is in accordance with the general trend in recent literature that weighted spaces are as necessary generalizations in current analysis of PDEs (Azzolini et al., 2024; Tian & Zheng, 2024). The theoretical and computational advantages obtained in this work encourage further investigation on weighted Sobolev formulations in nonlinear PDE problems of more general nature.

Future Direction

Future investigation extending these general existence and stability results to systems of nonlinear PDEs with dynamic boundary conditions and the presence of stochastic perturbations in weighted Sobolev spaces. Adaptive weight function schemes based on the solution feedback could be pursued to further improve computational techniques and controlling stability, which may lead towards the possibility to simulate in real-time the behavior of complex physical, biological systems and so on (Balao, Muñoz & Pastor, 2023; Fadil et al., 2024).

Limitations

The main limitation of this study is that our restriction to particular classes of nonlinear PDEs and fixed forms of weight function may not encompass all the diverse real-world problems with complex geometries or highly-irregular data. Also, the computational burden of the employed weighted norms is tolerable but still represent some difficulties for large scale simulations or high dimensional problems that would need to be addressed and optimized (Tian & Zheng, 2024; Srinivasan & Slotine, 2022).

Conclusion

The results of present paper provide theoretical and practical suggestion for using weighted Sobolev spaces in the study of generalized existence and stability of nonlinear partial differential equations. With state-of-the-art regularity estimates, stability, and numerical convergence, weighted spaces prove to be a versatile space for further development and applications in the theory of nonlinear PDEs. In addition to generalizing the classical analysis of the PDEs, the obtained results provide a valuable basis for future computational and practical work in complex nonlinear dynamical systems.

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