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SMART TECHNOLOGIES FOR WATER SEWAGE SYSTEMS AND DECISION-MAKING WITH CIRCULAR SPHERICAL FUZZY FRAMEWORK

Muhammad Ahmad

Department of Mathematics, Riphah International University (Lahore Campus), 54000, Lahore, Pakistan

ahmad341rai@gmail.com

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Corresponding Author: *
Muhammad Ahmad

Abstract
This work aims to improve decision-making (DM) processes by utilizing the circular spherical fuzzy set (Cr-SFS), a flexible structure for managing uncertain human opinions. This paper presents a new class of AOs, such as the circular spherical fuzzy Dombi weighted averaging (Cr-SFDWA), Circular spherical fuzzy Dombi weighted geometric (Cr-SFDWG), circular spherical fuzzy Dombi order weighted average (Cr-SFDOWA) and circular spherical fuzzy Dombi order weighted geometric (Cr-SFDOWG) operators which are specially designed for Cr-SF information systems. These operators' realistic qualities and exceptional cases are clarified, emphasizing how well they fit into real-world situations. A novel methodology for MADM is applied to various real-world applications with varying needs or features. An example of an AI selection process in a water sewage system is provided to show the effectiveness of the suggested methodologies. Moreover, a comprehensive comparison method is presented to illustrate the effectiveness and relevance of proposed aggregation strategies by comparing their outcomes with those of the existing approaches. The study is accomplished with a summary of its findings and a discussion of its prospects as we advance, highlighting the potential contribution of the suggested research to the advancement of decision-making techniques in dynamic and complex environments.

1. Introduction

Decision Making (DM) is selecting a single alternative from several alternatives to attain a desired result. It involves finding and assessing alternatives, considering the possible consequences, and choosing the most appropriate one based on criteria such as goals, values, and available information. The multiple-attribute decision-making (MADM) and multiple attribute group decision-making (MAGDM) are methodologies used in decision science for addressing problems involving numerous criteria or attributes. MADM involves DM, where innumerable criteria must be evaluated to choose the best alternative from several other options. It is used in scenarios where decisions must simultaneously consider various attributes or factors. The goal is to rank or select the most preferred alternative based on these attributes. Zhang and Ye [1] proposed the MADM model using Einstein aggregation operators of single-valued neutrosophic values and its application. MADM extends MAGDM by involving multiple decision-makers or stakeholders in the DM process. This approach is used when decisions need to be made collaboratively, considering the preferences and opinions of numerous individuals or groups. An extended CODAS method for MAGDM with 2-tuple linguistic t-spherical fuzzy sets (FS) was given by Akram et al. [2].

Zadeh [3] introduced the innovative theory of FS. According to Zadeh's FS theory, an element's membership grade (MG) determines whether or not it is a part of the set. In this instance, it is validated that the FS concept has boundless applicability. For example, Higashi and Klir et al.[4] works to an expansion of FS, uncertainty, and information which addresses almost every significant topic in the broad area of the union of FS theory and fuzzy logic. Goguen [5] explored the foundations of FS and continued the work of Zadeh. The theory of possibility presented by Zadeh [6] is related to FS theory by defining prospect sharing as a fuzzy constraint that serves as a flexible limit on the values that can be assigned to a variable.

With the help of an MG and non-membership grade (NMG), Atanassov [7] elaborated the concept of an intuitionistic fuzzy set (IFS), an accustomed shape of the FS that holds multifarious and unspecified facts. The IFS theory has proven to be quite effective and helpful in solving a wide range of issues. Despite its limitations, the IFS data is based on the irrational assumption that the sum of MG and NMG falls inside the interval [0 1]. Yager [8] proposed the idea of a pythagorean fuzzy set (PyFS), which is the improved version of IFS, to handle inaccurate and disorganized data. Like IFS, PyFS has a fussy range and is based on the situation that the total squares for MG and NMG stay in a stable assortment [0,1]. The MG and NMG cannot be observed as a Pythagorean fuzzy value if the total of the squares of the MG and NMG for a particular Pythagorean fuzzy value exceeds the interval$[0,1]$. For example, if MG is 0.5 and NMG is0.9. To solve the above issue, Yager [9] proposed the q-rung orthopair fuzzy set (Q-ROFS), a system that can handle uncertain and complex situations like the ones mentioned above. A pair of MG and NMG such that the sum of their respective q powers falls inside the unit interval is known as a q-rung orthopair fuzzy value. Using the parameter$q$, we can select any MG and NMG from the unit interval for each duplet (MG, NMG). The q is such that $0 \leq (MG)^q + (NMG)^q \leq 1$. In human opinion, there is also a certain amount of rejection and abstinence. This implies that the previous generalized form of FS structures could not handle these confusing errors.

Cuong [10] presented a picture fuzzy set (PFS) that consists of abstinence grade (AG), MG, and NMG to deal with these kinds of glitches, which are mentioned above. PFS's range also lies within unit intervals. In addition to the well-known FS, a unique type of FS named spherical fuzzy set (SFS) was recently introduced. SFS is the generalized structure over existing structures of FS (like IFS, PyFS,) based on three dimensions (truth, falsehood, and indeterminacy) to offer a broader choice for decision-making. Although the SFS has been presented lately, the topic attracts the devotion of academicians at a significant rate. Subsequently, Ashraf et al.[11] introduced the SFS. After making some variations, he presented the SFS by taking the qth. Gündoğdu and Kahraman [12] presented an overview of three-dimensional SFS containing some essential differences from the other FS. Spherical fuzzy distances serve as the foundation for this new kind of FS. Ozceylan et al. [13] presented the study's enlarged version, which is a thorough literature analysis of current and cutting-edge research done to create a framework for the past and provide guidance for the future. Torra and Godo [14] generalized three-dimensional SFS with their arithmetic, aggregation, and defuzzification operations. Akram. et al. [15] utilized an advanced DM technique of the TOPSIS method under the interval-valued spherical fuzzy environment system. Using the idea of SFS, Mehmood et al.[16] provided an approach for DM and medical diagnosis problems. The distance and similarity measurements for SFS and their applications in selecting mega projects were explained by Alreshidi et al. [17]. Helmers and Weiss [18] developed the idea of evaluating the life cycle of a battery using MADM. Lundström and Hellström [18] outlined how to broaden the app's evaluation capabilities for electric cars. Komunik et al. [19] compared the basic configuration of the exhaust emissions from vehicles running on unleaded gasoline and liquefied petroleum gas. Energies [21] presented a comparison of using a MADM-based traction supply system for charging electric vehicles. Vitta [22] elaborated the assessment of traditional fuel-powered vehicles with environmentally friendly features on MADM. The idea for the assortment of electric vehicles in MADM is the result of a careful and complex analysis given by Więckowski et al. [22]. Khan et al. [23] described the divergence measures for circular IFS (Cr-IFS) and their applications for engineering applications of artificial intelligence (AI). Atanassov and Marinov [24] presented the four distances for Cr-IFSs.

The concept of sustainable smart living, or SSL, is essential to the development of smart cities. Potential answers to urgent urban problems, including energy consumption, traffic congestion, and environmental degradation, can be found in these cities. Many SSL frameworks have been developed recently, taking into account various factors. However, All these problems cannot be addressed concurrently by any system. These are regarded as difficult assignments. Siksnelyte-Butkiene et al. [25] introduced multi-criteria decision-making (MCDM) and three-way decisions (TWD) as solutions to these problems, and these approaches can be helpful to decision-makers and city planners. In order to estimate the conditional probability of each alternative, this paper presents TWD-based MCDM as novel conditional probabilities by opinion scores (CPOS) within the framework of circular Py-FS (Cr-Py-FS) or the C-PFSs-CPOS technique. Lee and Yoon [26] developed the theory of AI based on technologies in the healthcare industry. Shan et al [27] give the concept of AI in dentistry. Zhou et al [28] used the application of AI in the field of surgery. Lawal and Kwon [29] developed the concept of AI in the field of rock mechanics. Wan et al. [30] extended the application of AI in computer network technology. Ghodousian et al. [31] proposed the concept of Sugeno-weber t-norm (TN) with the use of nonlinear optimization. M. Kauers et al. [32] developed the theory of Sugeno-Weber TN on FS. Farahbod and Eftekhari. [33] described the comparison of different TN operators in classification problems. Troiano et al. [34] compared the statistical analysis of parametric with Sugeno-Weber. Ghodousian. [35] introduced the concept of two fuzzy relational inequalities with Sugeno-Weber.

Despite the significant advancements in real-world environmental systems, there is a noticeable gap in the integration of sophisticated mathematical frameworks capable of addressing the uncertainties and complexities inherent in real-world environmental systems. Traditional DM approaches, including basic fuzzy logic and conventional aggregation operators (AOs), often fail to provide the necessary robustness and adaptability required for reliable outcomes in dynamic and uncertain conditions. While significant work has been done to enhance specific components of real-world environmental systems, such as the development of SFS and their applications in various DM scenarios by Mahmood et al. [16] and Ashraf et al. [10], there remains a critical gap in applying these advanced concepts to the real-world environmental systems. Specifically, few studies have explored the use of Cr-SF AOs, particularly those based on the Sugeno-Weber TN and t-conorm (TCN) within the framework of MADM for real-world environmental systems. Although the SFS framework has been explored in many domains, such as medical diagnosis Khan et al. [36] and mega project selection Alreshidi et al. [17], its application in environmental management systems, particularly in the context of intelligent water sewage systems, remains under-researched.

Moreover, the sensitivity of these operators to parameter variations, which is crucial for ensuring stability and flexibility in DM processes, has not been rigorously investigated. Existing research by Ghodousian et al. [35] on Sugeno-Weber TN and its applications in nonlinear optimization suggests potential benefits, but the specific use of Cr-SF AOs in this context requires further exploration. There is a clear need for research that not only introduces these advanced AOs to the field of water management but also thoroughly evaluates their performance, sensitivity, and reliability in practical applications. This study aims to address these gaps by applying Cr-SF AOs to the selection of intelligent water sewage systems, rigorously examining their sensitivity, and comparing their effectiveness against existing methods. By doing so, the research seeks to advance the field of environmental management, offering a more comprehensive and reliable decision-making framework for managing critical water infrastructure.

Effective and sustainable water resource management has become high on the list in the constant change of the contemporary world. Urbanization, population growth, and climate change put a strong pressure on existing water management systems and challenge available models to sustain effectiveness and reliability. Traditional methods of selection and system optimization have not succeeded in solving the problems and rising above the complexities and uncertainties confronting the issues. The next barrier to AI technology implementation is incorporating such intelligent systems into environmental management DM processes. This requires highly sophisticated tools that can address the acknowledged variations of uncertainty of actual cases. Especially on the basis of the Sugeno-Weber TN and TCN aggregation operators from Cr-SF, the door is opened to the stage.

This research is, therefore, motivated by the need to improve the frameworks for DM regarding the selection of intelligent water sewage systems. This paper attempts to apply the AOs of Cr-SF to devise a technique for the evaluation and optimum design of such important and sensitive infrastructure systems that are more robust, adaptable, and reliable in this regard. The objective is, therefore, to attempt to find a satisfactory solution within the context of multiple attributes, ensuring stability and flexibility for different scenarios with varying conditions and uncertainties. We hope that this work helps the decision-maker in this very hard arena of water resource management to be more confident and precise in decisions affecting society and the environment.

The outlines of this article are expressed as follows:

i. The circular spherical fuzzy framework emerges as an innovative extension of conventional and intuitionistic fuzzy sets to address more complex and uncertain decision-making environments. This framework provides a richer, more flexible representation of uncertainty, allowing decision-makers to express nuanced preferences and opinions.

ii. Integrating Dombi t-norms and t-conorms within the circular spherical fuzzy framework further refines its decision-making capabilities. Dombi t-norms, characterized by their parameterized flexibility, offer a customizable way to model the logical conjunction and disjunction operations.

iii. Based on distinct theories, we derived strong mathematical strategies from various theories, incorporating operators distinguished by their prominent characteristics and encompassing special cases such as circular spherical fuzzy Dombi weighted aggregation (Cr-SFDWA), circular spherical fuzzy Dombi weighted geometric (Cr-SFDWG), circular spherical fuzzy Dombi order weighted aggregation (Cr-SFDOWA), and circular spherical fuzzy Dombi order weighted geometric (Cr-SFDOWG).

iv. These derived strategies were employed to assess the effectiveness and applicability of newly developed AOs.

v. Additionally, we formulated a sophisticated DM method tailored to address stubborn and redundant human opinion data. Through the application of these methodologies, we conducted a comprehensive examination, including a numerical example, to evaluate the advanced technologies of sewage systems with numerical examples under decision-making problems.

The structure of the article is as follows: In section 2, we delve into the basic notions and principal rules for further development of new mathematical approaches. In section 3, we proposed a list of mathematical terminologies of Dombi weighted average operators with prominent properties. Section 4 also derives new geometric aggregation operators under consideration of Dombi t-norms and Cr-SF information. Section 5 proves the validation and reliability of proposed approaches with decision-making problems and experimental case studies of advanced sewage systems. Section 6 highlights the advantages and supremacy of pioneered approaches with existing mathematical terminologies. Concluding remarks and key findings are discussed in Section 7.

2. Preliminaries

In this section, we conducted a comprehensive overview of the Cr-SFSs with fundamental operations and Dombi t-norms.

Definition 1 [4] A PFS Z on non-empty set X is defined as:

$$Z = \{(\tau, \mu(\tau), v(\tau), \pi(\tau)) \mid \tau \in X\}$$

Noted that $\mu: X \to [0,1], v: X \to [0,1]$ and $\pi: X \to [0,1]$ indicate the MG, NMG and abstinence grade (AG) respectively with subject to condition $0 \leq \mu(\tau) + v(\tau) + \pi(\tau) \leq 1$. The refusal degree is denoted and defined by $\theta = 1 - (\mu(\tau) + v(\tau) + \pi(\tau))$.

Definition 2 [16] A SFS Z on a non-empty set X is defined as:

$$Z = \{(\tau, \mu(\tau), v(\tau), \pi(\tau)) \mid \tau \in X\}$$

Noted that $\mu: X \to [0,1], v: X \to [0,1]$ and $\pi: X \to [0,1]$ indicate the MG, NMG and AG, respectively with subject to condition $0 \leq \mu^2(\tau) + v^2(\tau) + \pi^2(\tau) \leq 1$. The refusal degree of τ is denoted by $\theta = \sqrt{1 - (\mu^2(\tau) + v^2(\tau) + \pi^2(\tau))}$.

Definition 3 [23] The shape of Cr-IFS Z on a non-empty set X is defined as follows:

$$Z = \{(\tau, \mu(\tau), \pi(\tau); r(\tau)) \mid \tau \in X\}$$

Where $\mu: X \to [0,1]$ and $\pi: X \to [0,1]$ representing the MG and NMG of an element τ in Z with subject to the condition $0 \leq \mu(\tau) + \pi(\tau) \leq 1$ and r is the radius of the circle for the element. Every element is validated by a circle with the center $(\mu(\tau), v(\tau))$ and radius $r \in [0,1]$ as opposed to the normal every point in the IFS shows an element in the intuitionistic fuzzy interpretation triangle.

Definition 4 [39] A Cr-PFS Y on a non-empty set X is defined by:

$$Y = \{(\tau, \mu(\tau), v(\tau), \pi(\tau); r(\tau)) \mid \tau \in X\}$$

Where $\mu: X \to [0,1], v: X \to [0,1]$ and $\pi: X \to [0,1]$ representing the MG, NMG and AG of an element τ in Y with subject to the condition $0 \leq \mu(\tau) + v(\tau) + \pi(\tau) \leq 1$ and $r \in [0,1]$ is the radius of the circle for elementτ. Furthermore, a Cr-PFV is denoted by $(\mu(\tau), v(\tau), \pi(\tau); r(\tau))$.

Definition 5 [40] A Cr-SFS Y on a non-empty set X is defined by:

$$Y = \{(\tau, \mu(\tau), v(\tau), \pi(\tau); r(\tau)) \mid \tau \in X\}$$

Where $\mu: X \to [0,1], v: X \to [0,1]$ and $\pi: X \to [0,1]$ representing the MG, NMG and AG of an element τ in Y with subject to the condition $0 \leq \mu^2(\tau) + v^2(\tau) + \pi^2(\tau) \leq 1$ and $r \in [0,1]$ is the radius of the circle for element τ. Furthermore, a Cr-SFV is denoted by $(\mu(\tau), v(\tau), \pi(\tau); r(\tau))$.

Definition 6 [40] Consider two Cr-SFV α_1 and β_1 in X:

$$\alpha_1 = \{(\tau, \mu_1(\tau), v_1(\tau), \pi_1(\tau); r_1(\tau)) \mid \tau \in X\}$$
$$\beta_1 = \{(\tau, \mu_2(\tau), v_2(\tau), \pi_2(\tau); r_2(\tau)) \mid \tau \in X\}$$

Given below are some operations on Cr-SFSs:

a) $\alpha_1 = \{(\tau, \mu_\alpha(\tau), v_\alpha(\tau), \pi_\alpha(\tau); r_1(\tau)) \mid \tau \in X\}$

b) $\alpha_1 \subset \beta_1$ iff $r_1 \leq r_2$ and $\mu_\alpha(\tau) \leq \mu_\beta(\tau)$, $v_\alpha(\tau) \leq v_\beta(\tau)$and $\pi_\alpha(\tau) \geq \pi_\beta(\tau)$

c) $\alpha_1 = \beta_1$ iff $r_1 = r_2$ and $\mu_\alpha(\tau) = \mu_\beta(\tau)$, $v_\alpha(\tau) = v_\beta(\tau)$and $\pi_\alpha(\tau) = \pi_\beta(\tau)$

d) $\alpha_1 \cup_{min} \beta_1 = \{\tau, \max(\mu_\alpha(\tau), \mu_\beta(\tau)), \min(v_\alpha(\tau), v_\beta(\tau)), min(\pi_\alpha(\tau), \pi_\beta(\tau)) \min(r_1\, r_2) \mid \tau \in X\}$

e) $\alpha_1 \cup_{max} \beta_1 = \{\tau, \max(\mu_\alpha(\tau), \mu_\beta(\tau)), \min(v_\alpha(\tau), v_\beta(\tau), min((\pi_\alpha(\tau), \pi_\beta(\tau))\max(r_1 r_2)) | \tau \in X\}$

f) $\alpha_1 \cap_{min} \beta_1 = \{(\tau, \min(\mu_\alpha(\tau), \mu_\beta(\tau)), \min(v_\alpha(\tau), v_\beta(\tau), \max(\pi_\alpha(\tau), \pi_\beta(\tau))\min(r_1, r_2))) | \tau \in X\}$

g) $\alpha_1 \cap_{max} \beta_1 = \{(\tau, \min(\mu_\alpha(\tau), \mu_\beta(\tau)), \min(v_\alpha(\tau), v_\beta(\tau), \max(\pi_\alpha(\tau), \pi_\beta(\tau)\max(r_1, r_2))) | \tau \in X\}$

Definition 7 [40] Assume $\alpha = (\mu_\alpha(\tau), v_\alpha(\tau), \pi_\alpha(\tau); r_\alpha(\tau))$ be a Cr-SFV and Score function is given by:

$$\mathbb{S}(\alpha) = \frac{1}{4}\left(\mu_\alpha^2(\tau) - v_\alpha^2(\tau) - \pi_\alpha^2(\tau) + \sqrt{2r}(2q - 1)\right)$$

Where $\mathbb{S}(\alpha) \in [1, -1]$ and $q \in [0,1]$

Accuracy function:

$$\propto(\alpha) = (\mu_\alpha(\tau))^2 + (v_\alpha(\tau))^2 + (\pi_\alpha(\tau))^2$$

Where $\propto(\alpha) \in [1, -1]$

Now Assume α *and* β be two Cr-SF systems then,

- If $\mathbb{S}(\alpha) > \mathbb{S}(\beta)$ then $\alpha > \beta$
- If $\mathbb{S}(\alpha) < \mathbb{S}(\beta)$ then $\alpha < \beta$

If $\mathbb{S}(\alpha) < \mathbb{S}(\beta)$ then

- $\propto(\alpha) > \propto(\beta)$ then $\alpha > \beta$
- $\propto(\alpha) < \propto(\beta)$ then $\alpha < \beta$
- $\propto(\alpha) = \propto(\beta)$ then $\alpha \approx \beta$

3. Circular Spherical Fuzzy Dombi Weighted Average Operators

Definition 8 Consider $\theta = (\mu(\tau), v(\tau), \pi(\tau); r(\tau)), \theta_1 = (\mu_1(\tau), v_1(\tau), \pi_1(\tau); r(\tau))$ and $\theta_2 = (\mu_2(\tau), v_{2(\tau)}, \pi_2(\tau); r(\tau))$ be any collection of Cr-SFVs. Then Dombi operational laws for Cr-SFVs based on TN and TCN are defined as:

1) $$\theta_1 \oplus_{min} \theta_2 = \begin{pmatrix} \sqrt{1 - \frac{1}{1+\left\{\left(\frac{\mu_1^2(\tau)}{1-\mu_1^2(\tau)}\right)^\gamma + \left(\frac{\mu_2^2(\tau)}{1-\mu_2^2(\tau)}\right)^\gamma\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\left(\frac{1-v_1^2(\tau)}{v_1^2(\tau)}\right)^\gamma + \left(\frac{1-v_2^2(\tau)}{v_2^2(\tau)}\right)^\gamma\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\left(\frac{1-\pi_1^2(\tau)}{\pi_1^2(\tau)}\right)^\gamma + \left(\frac{1-\pi_2^2(\tau)}{\pi_2^2(\tau)}\right)^\gamma\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\left(\frac{1-r_1^2(\tau)}{r_1^2(\tau)}\right)^\gamma + \left(\frac{1-r_2^2(\tau)}{r_2^2(\tau)}\right)^\gamma\right\}^{\frac{1}{\gamma}}}} \end{pmatrix}$$

2) $$\theta_1 \oplus_{max} \theta_2 \left(\begin{array}{c} \sqrt{1-\frac{1}{1+\left\{\left(\frac{\mu_1^2(\tau)}{1-\mu_1^2(\tau)}\right)^{\gamma}+\left(\frac{\mu_2^2(\tau)}{1-\mu_2^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\left(\frac{1-v_1^2(\tau)}{v_1^2(\tau)}\right)^{\gamma}+\left(\frac{1-v_2^2(\tau)}{v_2^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\left(\frac{1-\pi_1^2(\tau)}{\pi_1^2(\tau)}\right)^{\gamma}+\left(\frac{1-\pi_2^2(\tau)}{\pi_2^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{1-\frac{1}{1+\left\{\left(\frac{r_1^2(\tau)}{1-r_1^2(\tau)}\right)^{\gamma}+\left(\frac{r_2^2(\tau)}{1-r_2^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{array} \right)$$

3) $$\theta_1 \otimes_{min} \theta_2 = \left(\begin{array}{c} \sqrt{\frac{1}{1+\left\{\left(\frac{1-\mu_1^2(\tau)}{\mu_1^2(\tau)}\right)^{\gamma}+\left(\frac{1-\mu_2^2(\tau)}{\mu_2^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{1-\frac{1}{1+\left\{\left(\frac{v_1^2(\tau)}{1-v_1^2(\tau)}\right)^{\gamma}+\left(\frac{v_2^2(\tau)}{1-v_2^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{1-\frac{1}{1+\left\{\left(\frac{\pi_1^2(\tau)}{1-\pi_1^2(\tau)}\right)^{\gamma}+\left(\frac{\pi_2^2(\tau)}{1-\pi_2^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{1-\frac{1}{1+\left\{\left(\frac{r_1^2(\tau)}{1-r_1^2(\tau)}\right)^{\gamma}+\left(\frac{r_2^2(\tau)}{1-r_2^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{array} \right)$$

4) $$\theta_1 \otimes_{max} \theta_2 = \left(\begin{array}{c} \sqrt{\frac{1}{1+\left\{\left(\frac{1-\mu_1^2(\tau)}{\mu_1^2(\tau)}\right)^{\gamma}+\left(\frac{1-\mu_2^2(\tau)}{\mu_2^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{1-\frac{1}{1+\left\{\left(\frac{v_1^2(\tau)}{1-v_1^2(\tau)}\right)^{\gamma}+\left(\frac{v_2^2(\tau)}{1-v_2^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{1-\frac{1}{1+\left\{\left(\frac{\pi_1^2(\tau)}{1-\pi_1^2(\tau)}\right)^{\gamma}+\left(\frac{\pi_2^2(\tau)}{1-\pi_2^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\left(\frac{1-r_1^2(\tau)}{r_1^2(\tau)}\right)^{\gamma}+\left(\frac{1-r_2^2(\tau)}{r_2^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{array} \right)$$

5) $$\partial\theta_{min} = \begin{pmatrix} \sqrt{1-\frac{1}{1+\left\{\partial\left(\frac{\mu^2(\tau)}{1-\mu^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\partial\left(\frac{1-\nu^2(\tau)}{\nu^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\partial\left(\frac{1-\pi^2(\tau)}{\pi^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\partial\left(\frac{1-r^2(\tau)}{r^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{pmatrix}, \partial > 0$$

6) $$\partial\theta_{max} = \begin{pmatrix} \sqrt{1-\frac{1}{1+\left\{\partial\left(\frac{\mu^2(\tau)}{1-\mu^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\partial\left(\frac{1-\nu^2(\tau)}{\nu^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\partial\left(\frac{1-\pi^2(\tau)}{\pi^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{1-\frac{1}{1+\left\{\partial\left(\frac{r^2(\tau)}{1-r^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{pmatrix}, \partial > 0$$

7) $$\theta_{min}^{\partial} = \begin{pmatrix} \sqrt{\frac{1}{1+\left\{\partial\left(\frac{1-\mu^2(\tau)}{\mu^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{1-\frac{1}{1+\left\{\partial\left(\frac{1-\nu^2(\tau)}{\nu^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{1-\frac{1}{1+\left\{\partial\left(\frac{\pi^2(\tau)}{1-\pi^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{1-\frac{1}{1+\left\{\partial\left(\frac{1-r^2(\tau)}{r^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{pmatrix}, \partial > 0$$

8) $$\theta^{\partial}_{max}=\left(\begin{array}{c}\sqrt{\frac{1}{1+\left\{\partial\left(\frac{1-\mu^2(\tau)}{\mu^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{1-\frac{1}{1+\left\{\partial\left(\frac{v^2(\tau)}{1-v^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{1-\frac{1}{1+\left\{\partial\left(\frac{\pi^2(\tau)}{1-\pi^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{\partial\left(\frac{1-r^2(\tau)}{r^2(\tau)}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\end{array}\right),\partial>0$$

Definition 9 Let $\theta_h=\left(\mu_h(\tau),v_h(\tau),\pi_h(\tau);r_h(\tau)\right),(h=1,2,\dots,g)$ be a Collection of Cr-SFVs with $w=\left(w_1,w_2,\dots,w_g\right)^T$ be the weight assigned of θ_h such that $w_h\in[0,1]$ and $\sum_{h=1}^{g}w_h=1$. Then, the Cr-SFDWA operators are defined as follows:

$$Cr-SFDWA_{min}\left(\theta_1,\theta_2,\dots,\theta_g\right)=\oplus_{h=1}^{g}w_h\theta_h$$
$$Cr-SFDWA_{max}\left(\theta_1,\theta_2,\dots,\theta_g\right)=\oplus_{h=1}^{g}w_h\theta_h$$

Theorem 1 Let $\theta_h=\left(\mu_h(\tau),v_h(\tau),\pi_h(\tau);r_h(\tau)\right)(h=1,2,\dots,g)$ be a collection of Cr-SFVs. Then, the aggregated result by utilizing the discussed theory of the Cr-SFDWA operator gives again a Cr-SFV as follows:

$$Cr-SFDWA_{\min}\left(\theta_1,\theta_2,\dots,\theta_g\right)=\left(\begin{array}{c}\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{g}w_h\left(\frac{\mu_h^2}{1-\mu_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{g}w_h\left(\frac{1-v_h^2}{v_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{g}w_h\left(\frac{1-\pi_h^2}{\pi_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{g}w_h\left(\frac{1-r_h^2}{r_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\end{array}\right)$$

$$Cr-SFDWA_{max}(\theta_1,\theta_2,\dots,\theta_g)=\left(\begin{array}{c}\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{g} w_h\left(\frac{\mu_h^2}{1-\mu_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{g} w_h\left(\frac{1-v_h^2}{v_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{g} w_h\left(\frac{1-\pi_h^2}{\pi_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{g} w_h\left(\frac{r_h^2}{1-r_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\end{array}\right)$$

Proof: Based on Dombi operational laws, we have.

$$w_h\theta_h=\left(\begin{array}{c}\sqrt{1-\frac{1}{1+\left\{w_h\left(\frac{\mu_h^2}{1-\mu_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{w_h\left(\frac{1-v_h^2}{v_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \\ \sqrt{\frac{1}{1+\left\{w_h\left(\frac{1-\pi_h^2}{\pi_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{w_h\left(\frac{1-r_h^2}{r_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\end{array}\right)$$

We can prove the above the theorem by using mathematical induction.
If $h=2$, then from the above equation

$$w_1\theta_1 = \begin{pmatrix} \sqrt{1 - \dfrac{1}{1 + \left\{ w_1 \left(\frac{\mu_1^2}{1-\mu_1^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\dfrac{1}{1 + \left\{ w_1 \left(\frac{1-v_1^2}{v_1^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \\ \sqrt{\dfrac{1}{1 + \left\{ w_1 \left(\frac{1-\pi_1^2}{\pi_1^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\dfrac{1}{1 + \left\{ w_1 \left(\frac{1-r_1^2}{r_1^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \end{pmatrix}$$

and $$w_2\theta_2 = \begin{pmatrix} \sqrt{1 - \dfrac{1}{1 + \left\{ w_2 \left(\frac{\mu_2^2}{1-\mu_2^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\dfrac{1}{1 + \left\{ w_2 \left(\frac{1-v_2^2}{v_2^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \\ \sqrt{\dfrac{1}{1 + \left\{ w_2 \left(\frac{1-v_2^2}{v_2^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\dfrac{1}{1 + \left\{ w_2 \left(\frac{1-v_2^2}{v_2^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \end{pmatrix}$$

So $Cr - SFDWA(\theta_1, \theta_2) = w_1\theta_1 \oplus w_2\theta_2$

$$= \left[\begin{pmatrix} \sqrt{1 - \frac{1}{1+\left\{w_1\left(\frac{\mu_1^2}{1-\mu_1^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{w_1\left(\frac{1-v_1^2}{v_1^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \\ \sqrt{\frac{1}{1+\left\{w_1\left(\frac{1-\pi_1^2}{\pi_1^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{w_1\left(\frac{1-r_1^2}{r_1^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{pmatrix} \oplus \begin{pmatrix} \sqrt{1 - \frac{1}{1+\left\{w_2\left(\frac{\mu_2^2}{1-\mu_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{w_2\left(\frac{1-v_2^2}{v_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \\ \sqrt{\frac{1}{1+\left\{w_2\left(\frac{1-v_2^2}{v_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{w_2\left(\frac{1-v_2^2}{v_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{pmatrix}\right]$$

$$= \begin{pmatrix} \sqrt{1 - \frac{1}{1+\left\{\sum_{h=1}^{2} w_h\left(\frac{\mu_h^2}{1-\mu_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{2} w_h\left(\frac{1-v_h^2}{v_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{2} w_h\left(\frac{1-\pi_h^2}{\pi_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{2} w_h\left(\frac{1-r_h^2}{r_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{pmatrix}$$

Therefore, $$Cr - SFDWA(\theta_1, \theta_2) = \begin{pmatrix} \sqrt{1 - \frac{1}{1+\left\{\sum_{h=1}^{2} w_h\left(\frac{\mu_h^2}{1-\mu_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{2} w_h\left(\frac{1-v_h^2}{v_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{2} w_h\left(\frac{1-\pi_h^2}{\pi_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{2} w_h\left(\frac{1-r_h^2}{r_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{pmatrix}$$

Hence, it is valid for $h = 2$.
If $h = q$, then we get the following equation:

$$\mathrm{Cr}-\mathrm{SFDWA}(\theta_1,\theta_2)=\left(\begin{array}{c}\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{q} w_h\left(\frac{\mu_h^2}{1-\mu_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{q} w_h\left(\frac{1-v_h^2}{v_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{q} w_h\left(\frac{1-\pi_h^2}{\pi_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{q} w_h\left(\frac{1-r_h^2}{r_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\end{array}\right)$$

If $h = q + 1$, then from the above two equations, we have the following outcomes:
$Cr - SFDWA(\theta_1, \theta_2, \dots, \theta_{q+1}) = \oplus_{h=1}^{q} w_h \theta_k \oplus w_{q+1}\theta_{q+1}$;

$$=\left(\begin{array}{c}\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{q} w_h\left(\frac{\mu_h^2}{1-\mu_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{q} w_h\left(\frac{1-v_h^2}{v_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{q} w_h\left(\frac{1-\pi_h^2}{\pi_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{q} w_h\left(\frac{1-r_h^2}{r_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\end{array}\right)\oplus\left(\begin{array}{c}\sqrt{1-\frac{1}{1+\left\{w_{q+1}\left(\frac{\mu_{q+1}^2}{1-\mu_{q+1}^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{w_{q+1}\left(\frac{1-v_{q+1}^2}{v_{q+1}^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{w_{q+1}\left(\frac{1-\pi_{q+1}^2}{\pi_{q+1}^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{w_{q+1}\left(\frac{1-r_{q+1}^2}{r_{q+1}^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\end{array}\right)$$

$$= \begin{pmatrix} \sqrt{1 - \frac{1}{1+\left\{\sum_{h=1}^{q+1} w_h \left(\frac{\mu_h^2}{1-\mu_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{q+1} w_h \left(\frac{1-v_h^2}{v_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{q+1} w_h \left(\frac{1-\pi_h^2}{\pi_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{q+1} w_h \left(\frac{1-r_h^2}{r_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{pmatrix}$$

Thus, it is valid for all h. Hence, the Theorem is true.

Property 1: Let a family of Cr-SFVs $\theta_h = (\mu_h(\tau), v_h(\tau), \pi_h(\tau); r_h(\tau))(h = 1,2,\dots,g)$ are equal, that is $\theta_h = \theta$ for all h. then

$$Cr - SFDWA_{min}(\theta_1, \theta_2, \dots, \theta_g) = \theta$$
$$Cr - SFDWA_{max}(\theta_1, \theta_2, \dots, \theta_g) = \theta$$

Property 2: Let $\theta_h = \langle \mu_h(\tau), v_h(\tau), \pi_h(\tau); r_h(\tau) \rangle (h = 1,2,\dots,g)$ and $\hat{\theta} = (\widehat{\mu_h}(\tau), \widehat{v_h}(\tau), \widehat{\pi_h}(\tau); \widehat{r_h}(\tau))|\ (h = 1,2,\dots,g)$be a collection of two Cr-SFVs and $\theta_h \leq \hat{\theta}_h$ such that $\mu_h(\tau) \leq \widehat{\mu_h}(\tau), v_h(\tau) \geq \widehat{v_h}(\tau)\ and\ \pi_h(\tau) \geq \widehat{r_h}(\tau)$, for all h, Then:

$$Cr - SFDWA_{min}(\theta_1, \theta_2, \dots, \theta_g) \leq Cr - SFDWA_{min}(\widehat{\theta_1}, \widehat{\theta_2}, \dots, \widehat{\theta_g})$$
$$Cr - SFDWA_{max}(\theta_1, \theta_2, \dots, \theta_g) \leq Cr - SFDWA_{max}(\widehat{\theta_1}, \widehat{\theta_2}, \dots, \widehat{\theta_g})$$

Property 3 Let a family of Cr-SFVs $\theta_h = (\mu_h(\tau), v_h(\tau), \pi_h(\tau); r_h(\tau))(h = 1,2,\dots,g)$. If $(\theta_h)_{min} = (min\{\mu_h\}, max\{v_h\}, max\{\pi_h\})$, $(\theta_h)_{max} = (min\{\mu_h\}, max\{v_h\}, min\{\pi_h\})$ and $\theta_h = (\mu_h(\tau), v_h(\tau), \pi_h(\tau); r_h(\tau))(h = 1,2,\dots,g)$. If $(\theta_h^+)_{min} = (max\{\mu_h\}, min\{v_h\}, max\{\pi_h\})$, $(\theta_h^+)_{max} = (max\{\mu_h\}, min\{v_h\}, min\{\pi_h\})$. Then, the following axioms are defined as follows.

$$(\theta_h^-)_{min} \leq Cr - SFDWA_{min}(\theta_1, \theta_2, \dots, \theta_g) \leq (\theta_h^+)_{min}$$
$$(\theta_h^-)_{max} \leq Cr - SFDWA_{max}(\theta_1, \theta_2, \dots, \theta_g) \leq (\theta_h^+)_{max}$$

4. Circular Spherical Fuzzy Dombi Weighted Geometric Operators

Definition 10 Let $\theta_h = (\mu_h(\tau), v_h(\tau), \pi_h(\tau); r_h(\tau)), (h = 1,2,\dots,g)$ be a Collection of Cr-SFVs with $w = (w_1, w_2, \dots, w_g)^T$ be the weight vector of θ_h such that $w_h \in [0,1]$ and $\sum_{h=1}^{g} w_h = 1$. Then, the Cr-SFDWG operators are defined as follows:

$$Cr - SFDWG_{min}(\theta_1, \theta_2, \dots, \theta_g) = \otimes_{h=1}^{g} \theta_h^{w_h}$$
$$Cr - SFDWG_{max}(\theta_1, \theta_2, \dots, \theta_g) = \otimes_{h=1}^{g} \theta_h^{w_h}$$

Theorem 2 Consider $\theta_h = (\mu_h(\tau), v_h(\tau), \pi_h(\tau); r_h(\tau))(h = 1,2,\dots,g)$ be a collection of Cr-SFVs. Then, the aggregated result by utilizing the discussed theory of the Cr-SFDWG operators gives again a Cr-SFV as follows:

$$Cr-SFDWG(\theta_1,\theta_2,\dots,\theta_g)_{min}=\left(\begin{array}{c}\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{g}w_h\left(\frac{1-\mu_h^2}{\mu_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{g}w_h\left(\frac{1-v_h^2}{v_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{g}w_h\left(\frac{\pi_h^2}{1-\pi_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{g}w_h\left(\frac{1-r_h^2}{r_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\end{array}\right)$$

$$Cr-SFDWG_{max}(\theta_1,\theta_2,\dots,\theta_g)=\left(\begin{array}{c}\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{g}w_h\left(\frac{1-\mu_h^2}{\mu_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{\sum_{h=1}^{g}w_h\left(\frac{1-v_h^2}{v_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{g}w_h\left(\frac{\pi_h^2}{1-\pi_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\\ \sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{g}w_h\left(\frac{r_h^2}{1-r_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\end{array}\right)$$

Property 4 Let a family of Cr-SFVs $\theta_h=\big(\mu_h(\tau),v_h(\tau),\pi_h(\tau);r_h(\tau)\big)(h=1,2,\dots,g)$ are equal, that is $\theta_h=\theta$ for all h. then

$$Cr-SFDWG_{min}(\theta_1,\theta_2,\dots,\theta_g)=\theta$$
$$Cr-SFDWG_{max}(\theta_1,\theta_2,\dots,\theta_g)=\theta$$

Property 5 Let $\theta_h=\langle\mu_h(\tau),v_h(\tau),\pi_h(\tau);r_h(\tau)\rangle(h=1,2,\dots,g)$ and $\theta=\big(\widehat{\mu_h}(\tau),\widehat{v_h}(\tau),\widehat{\pi_h}(\tau);\widehat{r_h}(\tau)\big)|\ (h=1,2,\dots,g)$be a collection of two Cr-SFVs and $\theta_h\le\hat{\theta}_h$ such that $\mu_h(\tau)\le\widehat{\mu_h}(\tau),v_h(\tau)\ge\widehat{v_h}(\tau)\ and\ \pi_h(\tau)\ge\widehat{r_h}(\tau)$, for all h, Then

$$Cr-SFDWG_{min}(\theta_1,\theta_2,\dots,\theta_g)\le Cr-SFDWG_{min}(\widehat{\theta_1},\widehat{\theta_2},\dots,\widehat{\theta_g})$$
$$Cr-SFDWG_{max}(\theta_1,\theta_2,\dots,\theta_g)\le Cr-SFDWG_{max}(\widehat{\theta_1},\widehat{\theta_2},\dots,\widehat{\theta_g})$$

Property 6 Let a family of Cr-SFVs $\theta_h=\big(\mu_h(\tau),v_h(\tau),\pi_h(\tau);r_h(\tau)\big)(h=1,2,\dots,g)$. If $(\theta_h)_{min}=(min\{\mu_h\},max\{v_h\},max\{\pi_h\})$, $(\theta_h)_{max}=(min\{\mu_h\},max\{v_h\},min\{\pi_h\})$ and $\theta_h=\big(\mu_h(\tau),v_h(\tau),\pi_h(\tau);r_h(\tau)\big)(h=1,2,\dots,g)$. If $(\theta_h^{+})_{min}=(max\{\mu_h\},min\{v_h\},max\{\pi_h\})$, $(\theta_h^{+})_{max}=(max\{\mu_h\},min\{v_h\},min\{\pi_h\})$. Then, the following axioms are defined as follows.

$$(\theta_h^{-})_{min}\le Cr-SFDWG_{min}(\theta_1,\theta_2,\dots,\theta_g)\le(\theta_h^{+})_{min}$$

$$(\theta_h^-)_{max} \leq Cr - SFDWG_{max}(\theta_1, \theta_2, \dots, \theta_g) \leq (\theta_h^+)_{max}$$

5. Decision Algorithm of the MADM Problem for the Cr-SFS

Every aspect of life and various applications are included in the MADM problem, which considers different kinds of attribute information. Recently, many researchers have created mathematical techniques for dealing with complex numerical examples and real-life applications. We aim to provide more concrete aggregated results from imprecise and insufficient data, so we have created an effective method for solving the MADM problem. To serve this purpose, let a finite collection of an alternative denoted by $\not{C} = (\not{C}_1, \not{C}_2, \dots, \not{C}_n)$ and a set of finite attributes, $G = G_1, G_2, \dots, G_m$ associate degree of weight to each attribute $w = (w_1, w_2, \dots. w_j)$ such that $G_j > 0$. The decision proposed an innovative algorithm for the MADM technique as follows:

Step 1: The decision maker gets evidence about any realistic entity in the form of cr-SF information and states it as a standard decision matrix.

Step 2: Collected information having two types of attributes such as beneficial and non-beneficial. If given information contains more than one type of attribute, we need to transform the standard decision matrix into a normalized decision matrix:

$$\Re = (\theta_{jk})_{h\times g} = \begin{Bmatrix} (\mu_{jk}, v_{jk}, \pi_{jk}, r_{jk}) & for\ benificial\ C_k \\ (\pi_{jk}, v_{jk}, \mu_{jk}, r_{jk}) & for\ non - benificial\ C_k \end{Bmatrix}$$

Step 3: Applied derived approaches of the Cr-SFDWA and Cr-SFDWG operators.

$$Cr - SFDWA(\theta_{j1}, \theta_{j2}, \dots, \theta_{jg}) = \theta_{jk}$$
$$Cr - SFDWG(\theta_{j1}, \theta_{j2}, \dots, \theta_{jg}) = \theta_{jk}$$

Step 4: To identify practical optimal choices, the score values of each individual are thoroughly examined.

Step 5: The ranking and ordering technique is applied to reorganize all score values.

5.1 Experimental case study

Enhancing water sewage systems through AI technologies is crucial for the efficient management of water resources and environmental sustainability. These systems must possess capabilities such as projecting maintenance, adaptive resource allocation, real-time monitoring, and independent decision-making to address the difficulties of urban water management.

Five alternative AI technologies are identified for the development of an intelligent water sewage system:

Predictive Analytics $\not{C}_1$: Utilizing historical data and machine learning algorithms to predict equipment disasters and improve maintenance schedules in the sewage system.

Sensor Networks $\not{C}_2$: Deploying sensor networks for real-time monitoring of water quality, flow rates, and infrastructure conditions to detect leaks, blockages, or contamination events.

Optimization Algorithms $\not{C}_3$: Implementing optimization algorithms to dynamically allocate resources such as water flow, pressure, and treatment chemicals for efficient operation and energy conservation.

Autonomous Inspection Drones $\not{C}_4$: Utilizing autonomous drones equipped with cameras and sensors to inspect inaccessible or hazardous areas of the sewage system, providing visual data for analysis.

Remote Control Systems $\not{C}_5$: Developing remote control systems that enable operators to monitor and control sewage infrastructure remotely, facilitating quick response to emergencies or maintenance needs.

Now, let's define the attributes for evaluating these AI technologies in the context of the intelligent water sewage system:

Environmental Impact G_1: Assessing the extent to which the AI technology reduces water wastage, prevents pollution, and minimizes energy consumption in sewage treatment processes.

ReliabilityG_2: Evaluating the AI technology's ability to provide accurate and timely data for decision-making, ensuring the smooth operation of the sewage system without interruptions or errors.

Adaptability G_3: Measuring the AI technology's flexibility to adapt to changing environmental conditions, infrastructure configurations, and regulatory requirements in the water-sewage domain.

Cost effectiveness G_4: Analyzing the economic feasibility of implementing the AI technology, considering factors such as initial investment, operational costs, and long-term maintenance and resource utilization savings.

Scalability G_5: It refers to how well the AI technology can handle increased workloads or be expanded for larger systems. It measures if the AI can grow with rising data demands and if upgrading or integrating into bigger setups is easy.

The DM assigns weights to these attributes (0.25, 0.20, 0.30, 0.25), respectively, reflecting their relative importance in the evaluation process. Subsequently, the DM employs the prescribed methodologies to assess and select the most suitable AI technology for integration into the water sewage system. The evaluation process for different alternatives under the following steps of the MADM problem.

Step 1: The expert provides details about the alternatives based on the given attributes

Table 1: Details about the AI suppliers.

	G_1	G_2	G_3
$\not{C}_1$	$(0.28, 0.35, 0.47; 0.55)$	$(0.26, 0.33, 0.24; 0.33)$	$(0.15, 0.28, 0.44; 0.55)$
$\not{C}_2$	$(0.34, 0.88, 0.22; 0.34)$	$(0.17, 0.27, 0.47; 0.77)$	$(0.26, 0.33, 0.48; 0.49)$
$\not{C}_3$	$(0.55, 0.61, 0.33; 0.35)$	$(0.15, 0.18, 0.23; 0.24)$	$(0.49, 0.59, 0.16; 0.18)$
$\not{C}_4$	$(0.11, 0.15, 0.45; 0.55)$	$(0.44, 0.52, 0.12; 0.17)$	$(0.13, 0.18, 0.79; 0.82)$
$\not{C}_5$	$(0.21, 0.24, 0.37; 0.41)$	$(0.45, 0.48, 0.14; 0.17)$	$(0.45, 0.48, 0.17; 0.23)$
	G_4	G_5	
$\not{C}_1$	$(0.33, 0.46, 0.15; 0.17)$	$(0.15, 0.17, 0.46; 0.33)$	
$\not{C}_2$	$(0.21, 0.27, 0.14; 0193)$	$(0.14, 0.19, 0.21; 0.78)$	
$\not{C}_3$	$(0.47, 0.56, 0.11; 0.15)$	$(0.11, 0.15, 0.58; 0.44)$	
$\not{C}_4$	$(0.45, 0.56, 0.18; 0.24)$	$(0.18, 0.24, 0.40; 0.97)$	
$\not{C}_5$	$(0.15, 0.19, 0.12; 0.29)$	$(0.12, 0.29, 0.26; 0.86)$	

Step 2: Collected information having two types of attributes such as beneficial and non-beneficial. If given information contains more than one type of attribute, we need to convert the standard decision matrix into a normalized decision matrix; otherwise, this technique is necessary.

Step 3: Applied derived aggregation operators, and aggregated results are shown in Table 2. Table 2 shows the aggregated results for both operators, Cr-SFDWA and Cr-SFDWG. We also have the maximum and minimum values for both operators.

Table 2: Aggregated outcomes by the Cr-SFDWA and Cr-SFDWG operators.

	Cr-SFDWA (min)	Cr-SFDWA (max)
$\not{C}_1$	$(0.24107, 0.27611, 0.30132; 0.34555)$	$(0.24107, 0.27611, 0.30132; 0.47446)$
$\not{C}_2$	$(0.2607, 0.30564, 0.24222; 0.37451)$	$(0.2607, 0.30564, 0.24222; 0.61857)$
$\not{C}_3$	$(0.4482, 0.26613, 0.20071; 0.23142)$	$(0.4482, 0.26613, 0.20071; 0.30472)$
$\not{C}_4$	$(0.27505, 0.19792, 0.23247; 0.32353)$	$(0.27505, 0.19792, 0.23247; 0.86786)$
$\not{C}_5$	$(0.33738, 0.28897, 0.18487; 0.26491)$	$(0.33738, 0.28897, 0.18487; 0.5846)$
	Cr-SFDWG (min)	Cr-SFDWG (max)

$₵_1$	(0.19826, 0.27611, 0.41215; 0.34555)	(0.19826, 0.27611, 0.41215; 0.47446)
$₵_2$	(0.21028, 0.30564, 0.35676; 0.37451)	(0.21028, 0.30564, 0.35676; 0.61857)
$₵_3$	(0.20875, 0.26613, 0.34091; 0.23142)	(0.20875, 0.26613, 0.34091; 0.30472)
$₵_4$	(0.14541, 0.19792, 0.58889; 0.32353)	(0.14541, 0.19792, 0.58889; 0.86786)
$₵_5$	(0.20311, 0.28897, 0.26339; 0.26491)	(0.20311, 0.28897, 0.26339; 0.5846)

Step 4: In order to identify practical optimal choices, the score values of each individual are thoroughly examined.

Table 3: Score values corresponding to each alternative.

	Cr-SFDWA (min)	Cr-SFDWA (max)	Cr-SFDWG (min)	Cr-SFDWG (max)
$₵_1$	0.1598	0.1919	0.1353	0.1675
$₵_2$	0.1737	0.2292	0.1506	0.1737
$₵_3$	0.1756	0.1982	0.1172	0.2061
$₵_4$	0.1766	0.2920	0.0898	0.1398
$₵_5$	0.1628	0.2423	0.1359	0.2154

Step 5: Estimate score values of all preferences and state them in Table 4.

Table 4: Ranking of alternatives based on score values.

Aggregation Operators	Ranking of preferences
Cr-SFDWA (min)	$₵_4 \succ ₵_3 \succ ₵_2 \succ ₵_5 \succ ₵_1$
Cr-SFDWA (max)	$₵_4 \succ ₵_5 \succ ₵_2 \succ ₵_3 \succ ₵_1$
Cr-SFDWG (min)	$₵_2 \succ ₵_5 \succ ₵_1 \succ ₵_3 \succ ₵_4$
Cr-SFDWG (max)	$₵_5 \succ ₵_2 \succ ₵_4 \succ ₵_1 \succ ₵_3$

Figure 1 makes it easier to understand the trends of the calculated and aggregated results in Table 4.

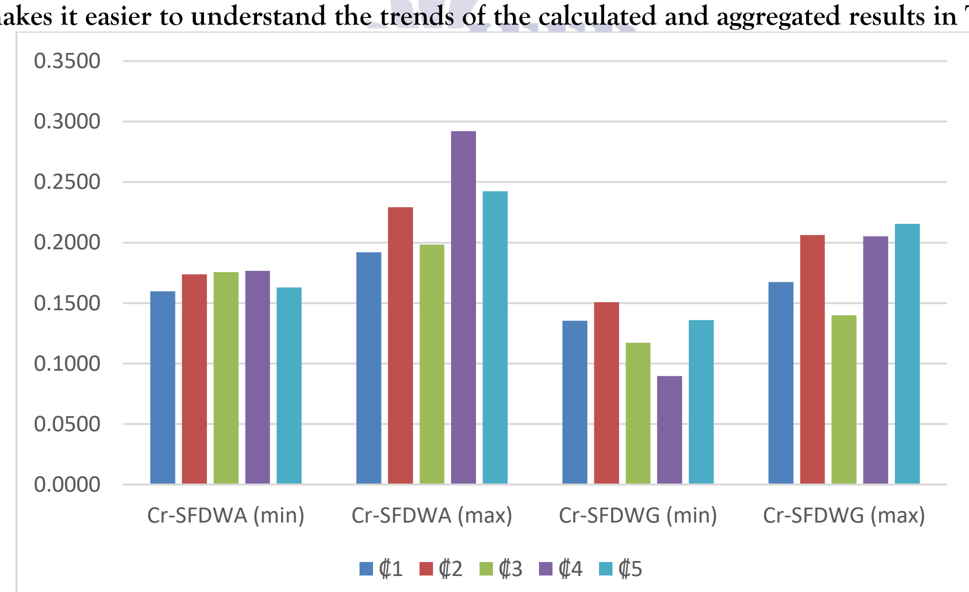

Figure 1 shows aggregated outcomes by the derived approaches.

The bar chart illustrates the comparative performance of five series across four different aggregation operators: Cr-SFDWA (min), Cr-SFDWA (max), Cr-SFDWG (min), and Cr-SFDWG (max). The values on the y-axis represent the aggregation results, ranging from 0.0000 to 0.3500. All series have similar values for the Cr-SFDWA (min) operator, with a slight variation, indicating relatively consistent performance across the series. In the Cr-SFDWA (max) category, Series 4 stands out with the highest value, indicating a significantly better performance compared to the other series. Series 5 and Series 2 also show higher values in this category. In the Cr-SFDWG (min) category, Series 1, Series 2, and Series 5 display similar results, while Series 3 and Series 4 have lower values, suggesting weaker performance. Finally, in the Cr-SFDWG (max) category, the values across all series are more varied, with Series 2 and Series 5 showing higher results, while Series 3 has the lowest.

Overall, the chart reveals differences in performance across the various aggregation operators, with some series consistently performing better than others in certain categories. Series 4 shows notable variation between the min and max categories, especially in the Cr-SFDWA operator.

5.2 Sensitivity analysis

The sensitivity analysis in this study plays a key role in checking how reliable the aggregation operators (Cr-SFDWA, Cr-SFDWG, Cr-SFDOWA, and Cr-SFDOWG) are, especially when choosing the best intelligent water sewage system. By changing the Dombi TN and TCN parameters from $\mathcal{J} = 1$ to $\mathcal{J} = 100$, we were able to see how these changes affect the rankings of the different options, $\not{C}_1$ to $\not{C}_5$.

The Cr-SFDPWA operator stood out for its stability, consistently keeping the same ranking order of $\not{C}_5 \succ \not{C}_2 \succ \not{C}_1 \succ \not{C}_4 \succ \not{C}_3$ no matter how the parameters were adjusted. This shows that the Cr-SFDWA operator is not easily influenced by changes, making it a reliable choice when stability in decision-making is crucial. The Cr-SFDOWA operator also showed similar consistency, making it a strong option for situations where dependable decision-making is important.

Table 5: Results obtained from the $Cr - SFDWA_{min}$ operator by varying the values of $\mathcal{J}$.

	Score Values	Ranking of alternatives
$\mathcal{J} = 1$	$\mathbb{S}(\not{C}_1) = 0.1598, \mathbb{S}(\not{C}_2) = 0.1737, \mathbb{S}(\not{C}_3) = 0.1756, \mathbb{S}(\not{C}_4) = 0.1766, \mathbb{S}(\not{C}_5) = 0.1698$	$\not{C}_4 \succ \not{C}_3 \succ \not{C}_2 \succ \not{C}_5 \succ \not{C}_1$
$\mathcal{J} = 2$	$\mathbb{S}(\not{C}_1) = 0.1546, \mathbb{S}(\not{C}_2) = 0.1667, \mathbb{S}(\not{C}_3) = 0.1854, \mathbb{S}(\not{C}_4) = 0.1714, \mathbb{S}(\not{C}_5) = 0.1655$	$\not{C}_3 \succ \not{C}_4 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\mathcal{J} = 5$	$\mathbb{S}(\not{C}_1) = 0.1470, \mathbb{S}(\not{C}_2) = 0.1581, \mathbb{S}(\not{C}_3) = 0.1889, \mathbb{S}(\not{C}_4) = 0.1705, \mathbb{S}(\not{C}_5) = 0.1677$	$\not{C}_3 \succ \not{C}_4 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\mathcal{J} = 20$	$\mathbb{S}(\not{C}_1) = 0.1453, \mathbb{S}(\not{C}_2) = 0.1547, \mathbb{S}(\not{C}_3) = 0.1898, \mathbb{S}(\not{C}_4) = 0.1711, \mathbb{S}(\not{C}_5) = 0.1688$	$\not{C}_3 \succ \not{C}_4 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\mathcal{J} = 35$	$\mathbb{S}(\not{C}_1) = 0.1454, \mathbb{S}(\not{C}_2) = 0.1543, \mathbb{S}(\not{C}_3) = 0.1900, \mathbb{S}(\not{C}_4) = 0.1715, \mathbb{S}(\not{C}_5) = 0.1690$	$\not{C}_3 \succ \not{C}_4 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\mathcal{J} = 50$	$\mathbb{S}(\not{C}_1) = 0.1454, \mathbb{S}(\not{C}_2) = 0.1541, \mathbb{S}(\not{C}_3) = 0.1901, \mathbb{S}(\not{C}_4) = 0.1717, \mathbb{S}(\not{C}_5) = 0.1690$	$\not{C}_3 \succ \not{C}_4 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\mathcal{J} = 65$	$\mathbb{S}(\not{C}_1) = 0.1455, \mathbb{S}(\not{C}_2) = 0.1540, \mathbb{S}(\not{C}_3) = 0.1901, \mathbb{S}(\not{C}_4) = 0.1719, \mathbb{S}(\not{C}_5) = 0.1691$	$\not{C}_3 \succ \not{C}_4 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\mathcal{J} = 75$	$\mathbb{S}(\not{C}_1) = 0.1455, \mathbb{S}(\not{C}_2) = 0.1539, \mathbb{S}(\not{C}_3) = 0.1901, \mathbb{S}(\not{C}_4) = 0.1719, \mathbb{S}(\not{C}_5) = 0.1691$	$\not{C}_3 \succ \not{C}_4 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\mathcal{J} = 90$	$\mathbb{S}(\not{C}_1) = 0.1455, \mathbb{S}(\not{C}_2) = 0.1539, \mathbb{S}(\not{C}_3) = 0.1901, \mathbb{S}(\not{C}_4) = 0.1720, \mathbb{S}(\not{C}_5) = 0.1691$	$\not{C}_3 \succ \not{C}_4 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$
$\mathcal{J} = 100$	$\mathbb{S}(\not{C}_1) = 0.1455, \mathbb{S}(\not{C}_2) = 0.1539, \mathbb{S}(\not{C}_3) = 0.1901, \mathbb{S}(\not{C}_4) = 0.1721, \mathbb{S}(\not{C}_5) = 0.1691$	$\not{C}_3 \succ \not{C}_4 \succ \not{C}_5 \succ \not{C}_2 \succ \not{C}_1$

On the other hand, the Cr-SFDWG and Cr-SFDOWG operators were a bit more sensitive to changes in the parameters. While the overall rankings remained stable, there were some minor shifts in the scores. This slight sensitivity adds a level of flexibility, allowing for more detailed decision-making. These operators are particularly useful in situations where some adaptability is needed.

In conclusion, the sensitivity analysis shows that the Cr-SFDWA and Cr-SFDOWA operators are the most reliable for making consistent decisions. Meanwhile, the Cr-SFDWG and Cr-SFDOWG operators, though slightly more sensitive, provide useful flexibility for decisions that require some level of adaptability. This analysis highlights how effective these new Cr-SF operators are in offering both reliable and flexible solutions for selecting intelligent water sewage systems. This version is designed to be more approachable and easier to understand while still conveying the essential points of your analysis.

Table 6: Results obtained from the $\boldsymbol{Cr-SFDWA_{max}}$ operator by varying the values of $\boldsymbol{\mathcal{J}}$.

	Score Values	Ranking of alternatives
$\mathcal{J}=1$	$\mathbb{S}(Ȼ_1)=0.1919, \mathbb{S}(Ȼ_2)=0.2292, \mathbb{S}(Ȼ_3)=0.1982, \mathbb{S}(Ȼ_4)=0.2920, \mathbb{S}(Ȼ_5)=0.2423$	$Ȼ_4 \succ Ȼ_5 \succ Ȼ_2 \succ Ȼ_3 \succ Ȼ_1$
$\mathcal{J}=2$	$\mathbb{S}(Ȼ_1)=0.2114, \mathbb{S}(Ȼ_2)=0.2536, \mathbb{S}(Ȼ_3)=0.2240, \mathbb{S}(Ȼ_4)=0.3194, \mathbb{S}(Ȼ_5)=0.2827$	$Ȼ_4 \succ Ȼ_5 \succ Ȼ_2 \succ Ȼ_3 \succ Ȼ_1$
$\mathcal{J}=5$	$\mathbb{S}(Ȼ_1)=0.2318, \mathbb{S}(Ȼ_2)=0.2769, \mathbb{S}(Ȼ_3)=0.2495, \mathbb{S}(Ȼ_4)=0.3393, \mathbb{S}(Ȼ_5)=0.3124$	$Ȼ_4 \succ Ȼ_5 \succ Ȼ_2 \succ Ȼ_3 \succ Ȼ_1$
$\mathcal{J}=20$	$\mathbb{S}(Ȼ_1)=0.2452, \mathbb{S}(Ȼ_2)=0.2909, \mathbb{S}(Ȼ_3)=0.2701, \mathbb{S}(Ȼ_4)=0.3501, \mathbb{S}(Ȼ_5)=0.3280$	$Ȼ_4 \succ Ȼ_5 \succ Ȼ_2 \succ Ȼ_3 \succ Ȼ_1$
$\mathcal{J}=35$	$\mathbb{S}(Ȼ_1)=0.2474, \mathbb{S}(Ȼ_2)=0.2930, \mathbb{S}(Ȼ_3)=0.2735, \mathbb{S}(Ȼ_4)=0.3518, \mathbb{S}(Ȼ_5)=0.3301$	$Ȼ_4 \succ Ȼ_5 \succ Ȼ_2 \succ Ȼ_3 \succ Ȼ_1$
$\mathcal{J}=50$	$\mathbb{S}(Ȼ_1)=0.2483, \mathbb{S}(Ȼ_2)=0.2938, \mathbb{S}(Ȼ_3)=0.2749, \mathbb{S}(Ȼ_4)=0.3526, \mathbb{S}(Ȼ_5)=0.3311$	$Ȼ_4 \succ Ȼ_5 \succ Ȼ_2 \succ Ȼ_3 \succ Ȼ_1$
$\mathcal{J}=65$	$\mathbb{S}(Ȼ_1)=0.2487, \mathbb{S}(Ȼ_2)=0.2943, \mathbb{S}(Ȼ_3)=0.2756, \mathbb{S}(Ȼ_4)=0.3531, \mathbb{S}(Ȼ_5)=0.3316$	$Ȼ_4 \succ Ȼ_5 \succ Ȼ_2 \succ Ȼ_3 \succ Ȼ_1$
$\mathcal{J}=75$	$\mathbb{S}(Ȼ_1)=0.2490, \mathbb{S}(Ȼ_2)=0.2945, \mathbb{S}(Ȼ_3)=0.2759, \mathbb{S}(Ȼ_4)=0.3533, \mathbb{S}(Ȼ_5)=0.3318$	$Ȼ_4 \succ Ȼ_5 \succ Ȼ_2 \succ Ȼ_3 \succ Ȼ_1$
$\mathcal{J}=90$	$\mathbb{S}(Ȼ_1)=0.2492, \mathbb{S}(Ȼ_2)=0.2948, \mathbb{S}(Ȼ_3)=0.2763, \mathbb{S}(Ȼ_4)=0.3535, \mathbb{S}(Ȼ_5)=0.3320$	$Ȼ_4 \succ Ȼ_5 \succ Ȼ_2 \succ Ȼ_3 \succ Ȼ_1$
$\mathcal{J}=100$	$\mathbb{S}(Ȼ_1)=0.2493, \mathbb{S}(Ȼ_2)=0.2949, \mathbb{S}(Ȼ_3)=0.2765, \mathbb{S}(Ȼ_4)=0.3537, \mathbb{S}(Ȼ_5)=0.3321$	$Ȼ_4 \succ Ȼ_5 \succ Ȼ_2 \succ Ȼ_3 \succ Ȼ_1$

Table 7: Results obtained from the $\boldsymbol{Cr-SFDWG_{min}}$ operator by varying the values of $\boldsymbol{\mathcal{J}}$.

	Score Values	Ranking of alternatives
$\mathcal{J}=1$	$\mathbb{S}(Ȼ_1)=0.1353, \mathbb{S}(Ȼ_2)=0.1506, \mathbb{S}(Ȼ_3)=0.1172, \mathbb{S}(Ȼ_4)=0.0898, \mathbb{S}(Ȼ_5)=0.1359$	$Ȼ_2 \succ Ȼ_5 \succ Ȼ_1 \succ Ȼ_3 \succ Ȼ_4$
$\mathcal{J}=2$	$\mathbb{S}(Ȼ_1)=0.1150, \mathbb{S}(Ȼ_2)=0.1280, \mathbb{S}(Ȼ_3)=0.0981, \mathbb{S}(Ȼ_4)=0.0402, \mathbb{S}(Ȼ_5)=0.1218$	$Ȼ_5 \succ Ȼ_2 \succ Ȼ_1 \succ Ȼ_3 \succ Ȼ_4$
$\mathcal{J}=5$	$\mathbb{S}(Ȼ_1)=0.0927, \mathbb{S}(Ȼ_2)=0.1005, \mathbb{S}(Ȼ_3)=0.0686, \mathbb{S}(Ȼ_4)=-0.0001, \mathbb{S}(Ȼ_5)=0.1058$	$Ȼ_5 \succ Ȼ_2 \succ Ȼ_1 \succ Ȼ_3 \succ Ȼ_4$
$\mathcal{J}=20$	$\mathbb{S}(Ȼ_1)=0.0797, \mathbb{S}(Ȼ_2)=0.0834, \mathbb{S}(Ȼ_3)=0.0461, \mathbb{S}(Ȼ_4)=-0.0207, \mathbb{S}(Ȼ_5)=0.0952$	$Ȼ_5 \succ Ȼ_2 \succ Ȼ_1 \succ Ȼ_3 \succ Ȼ_4$
$\mathcal{J}=35$	$\mathbb{S}(Ȼ_1)=0.0775, \mathbb{S}(Ȼ_2)=0.0807, \mathbb{S}(Ȼ_3)=0.0425, \mathbb{S}(Ȼ_4)=-0.0236, \mathbb{S}(Ȼ_5)=0.0937$	$Ȼ_5 \succ Ȼ_2 \succ Ȼ_1 \succ Ȼ_3 \succ Ȼ_4$
$\mathcal{J}=50$	$\mathbb{S}(Ȼ_1)=0.0766, \mathbb{S}(Ȼ_2)=0.0796, \mathbb{S}(Ȼ_3)=0.0410, \mathbb{S}(Ȼ_4)=-0.0248, \mathbb{S}(Ȼ_5)=0.0930$	$Ȼ_5 \succ Ȼ_2 \succ Ȼ_1 \succ Ȼ_3 \succ Ȼ_4$
$\mathcal{J}=65$	$\mathbb{S}(Ȼ_1)=0.0761, \mathbb{S}(Ȼ_2)=0.0790, \mathbb{S}(Ȼ_3)=0.0402,$	$Ȼ_5 \succ Ȼ_2 \succ Ȼ_1 \succ Ȼ_3 \succ Ȼ_4$

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	$\mathbb{S}(Ȼ_4) = -0.0254, \mathbb{S}(Ȼ_5) = 0.0927$	
$\mathcal{J} = 75$	$\mathbb{S}(Ȼ_1) = 0.0759, \mathbb{S}(Ȼ_2) = 0.0787, \mathbb{S}(Ȼ_3) = 0.0399,$ $\mathbb{S}(Ȼ_4) = -0.0257, \mathbb{S}(Ȼ_5) = 0.0925$	$Ȼ_5 \succ Ȼ_2 \succ Ȼ_1 \succ Ȼ_3 \succ Ȼ_4$
$\mathcal{J} = 90$	$\mathbb{S}(Ȼ_1) = 0.0756, \mathbb{S}(Ȼ_2) = 0.0784, \mathbb{S}(Ȼ_3) = 0.0395,$ $\mathbb{S}(Ȼ_4) = -0.0259, \mathbb{S}(Ȼ_5) = 0.0924$	$Ȼ_5 \succ Ȼ_2 \succ Ȼ_1 \succ Ȼ_3 \succ Ȼ_4$
$\mathcal{J} = 100$	$\mathbb{S}(Ȼ_1) = 0.0755, \mathbb{S}(Ȼ_2) = 0.0783, \mathbb{S}(Ȼ_3) = 0.0393,$ $\mathbb{S}(Ȼ_4) = -0.0261, \mathbb{S}(Ȼ_5) = 0.0923$	$Ȼ_5 \succ Ȼ_2 \succ Ȼ_1 \succ Ȼ_3 \succ Ȼ_4$

Table 8: Results obtained from the $\boldsymbol{Cr - SFDWG_{max}}$ operator by varying the values of $\boldsymbol{\mathcal{J}}$.

	Score Values	Ranking of alternatives
$\mathcal{J} = 1$	$\mathbb{S}(Ȼ_1) = 0.1675, \mathbb{S}(Ȼ_2) = 0.2061, \mathbb{S}(Ȼ_3) = 0.1398,$ $\mathbb{S}(Ȼ_4) = 0.2052, \mathbb{S}(Ȼ_5) = 0.2154$	$Ȼ_5 \succ Ȼ_2 \succ Ȼ_1 \succ Ȼ_4 \succ Ȼ_3$
$\mathcal{J} = 2$	$\mathbb{S}(Ȼ_1) = 0.1718, \mathbb{S}(Ȼ_2) = 0.2150, \mathbb{S}(Ȼ_3) = 0.1366,$ $\mathbb{S}(Ȼ_4) = 0.1883, \mathbb{S}(Ȼ_5) = 0.2391$	$Ȼ_5 \succ Ȼ_2 \succ Ȼ_1 \succ Ȼ_4 \succ Ȼ_3$
$\mathcal{J} = 5$	$\mathbb{S}(Ȼ_1) = 0.1774, \mathbb{S}(Ȼ_2) = 0.2193, \mathbb{S}(Ȼ_3) = 0.1292,$ $\mathbb{S}(Ȼ_4) = 0.16883, \mathbb{S}(Ȼ_5) = 0.2506$	$Ȼ_5 \succ Ȼ_2 \succ Ȼ_1 \succ Ȼ_4 \succ Ȼ_3$
$\mathcal{J} = 20$	$\mathbb{S}\mathbb{S}(Ȼ_1) = 0.1774, \mathbb{S}(Ȼ_2) = 0.2196, \mathbb{S}(Ȼ_3) = 0.1264,$ $\mathbb{S}(Ȼ_4) = 0.1583, \mathbb{S}(Ȼ_5) = 0.2546$	$Ȼ_5 \succ Ȼ_2 \succ Ȼ_1 \succ Ȼ_4 \succ Ȼ_3$
$\mathcal{J} = 35$	$\mathbb{S}(Ȼ_1) = 0.1795, \mathbb{S}(Ȼ_2) = 0.2194, \mathbb{S}(Ȼ_3) = 0.1260,$ $\mathbb{S}(Ȼ_4) = 0.1568, \mathbb{S}(Ȼ_5) = 0.2549$	$Ȼ_5 \succ Ȼ_2 \succ Ȼ_1 \succ Ȼ_4 \succ Ȼ_3$
$\mathcal{J} = 50$	$\mathbb{S}(Ȼ_1) = 0.1795, \mathbb{S}(Ȼ_2) = 0.2193, \mathbb{S}(Ȼ_3) = 0.1258,$ $\mathbb{S}(Ȼ_4) = 0.1562, \mathbb{S}(Ȼ_5) = 0.2551$	$Ȼ_5 \succ Ȼ_2 \succ Ȼ_1 \succ Ȼ_4 \succ Ȼ_3$
$\mathcal{J} = 65$	$\mathbb{S}(Ȼ_1) = 0.1794, \mathbb{S}(Ȼ_2) = 0.2193, \mathbb{S}(Ȼ_3) = 0.1258,$ $\mathbb{S}(Ȼ_4) = 0.1558, \mathbb{S}(Ȼ_5) = 0.2552$	$Ȼ_5 \succ Ȼ_2 \succ Ȼ_1 \succ Ȼ_4 \succ Ȼ_3$
$\mathcal{J} = 75$	$\mathbb{S}(Ȼ_1) = 0.1794, \mathbb{S}(Ȼ_2) = 0.2193, \mathbb{S}(Ȼ_3) = 0.1257,$ $\mathbb{S}(Ȼ_4) = 0.1557, \mathbb{S}(Ȼ_5) = 0.2552$	$Ȼ_5 \succ Ȼ_2 \succ Ȼ_1 \succ Ȼ_4 \succ Ȼ_3$
$\mathcal{J} = 90$	$\mathbb{S}(Ȼ_1) = 0.1793, \mathbb{S}(Ȼ_2) = 0.2193, \mathbb{S}(Ȼ_3) = 0.1257,$ $\mathbb{S}(Ȼ_4) = 0.1555, \mathbb{S}(Ȼ_5) = 0.2552$	$Ȼ_5 \succ Ȼ_2 \succ Ȼ_1 \succ Ȼ_4 \succ Ȼ_3$
$\mathcal{J} = 100$	$\mathbb{S}(Ȼ_1) = 0.1793, \mathbb{S}(Ȼ_2) = 0.2193, \mathbb{S}(Ȼ_3) = 0.1257,$ $\mathbb{S}(Ȼ_4) = 0.1555, \mathbb{S}(Ȼ_5) = 0.2553$	$Ȼ_5 \succ Ȼ_2 \succ Ȼ_1 \succ Ȼ_4 \succ Ȼ_3$

It's clear that $Ȼ_5$ is the best alternative in all of them because 5 has the higher score value in all different operators. So, we find the result that Remote Control Systems $Ȼ_5$ is the best function of AI.

6. Comparative Analysis

In this section, we offer a comparative analysis of our proposed work with the previously existing techniques. In recent years, the selection of optimal systems has been addressed through various methodologies, each leveraging different mathematical and computational frameworks.

We use data from Tables 5, 6, 7 and 8 to compare a subset of modern operators with the pioneering operators. By comparing the unique and current techniques to improve the value and capacity of the diagnosed operators, we were able to demonstrate the significant improvement of the derived work over the current operators with multiple demonstrative particle examples. We use the work of Ashraf et al. [10] and compare it with our aggregated result.

Table 9: demonstrate results of the comparative study.

Operators	Score Values
Cr-SFDWA (min)	$Ȼ_4 \succ Ȼ_3 \succ Ȼ_2 \succ Ȼ_5 \succ Ȼ_1$
Cr-SFDWA (max)	$Ȼ_4 \succ Ȼ_5 \succ Ȼ_2 \succ Ȼ_3 \succ Ȼ_1$

Cr-SFDWG (min)	$₵_2 \succ ₵_5 \succ ₵_1 \succ ₵_3 \succ ₵_4$
Cr-SFDWG (max)	$₵_5 \succ ₵_2 \succ ₵_4 \succ ₵_1 \succ ₵_3$
Cr-SWWA (min) [10]	$₵_2 \succ ₵_1 \succ ₵_5 \succ ₵_3 \succ ₵_4$
Cr-SFSWA (max) [10]	$₵_3 \succ ₵_5 \succ ₵_4 \succ ₵_2 \succ ₵_1$
Cr-SFSWG (min) [10]	$₵_2 \succ ₵_5 \succ ₵_1 \succ ₵_3 \succ ₵_4$
Cr-SFSWG (max) [10]	$₵_3 \succ ₵_4 \succ ₵_5 \succ ₵_2 \succ ₵_1$

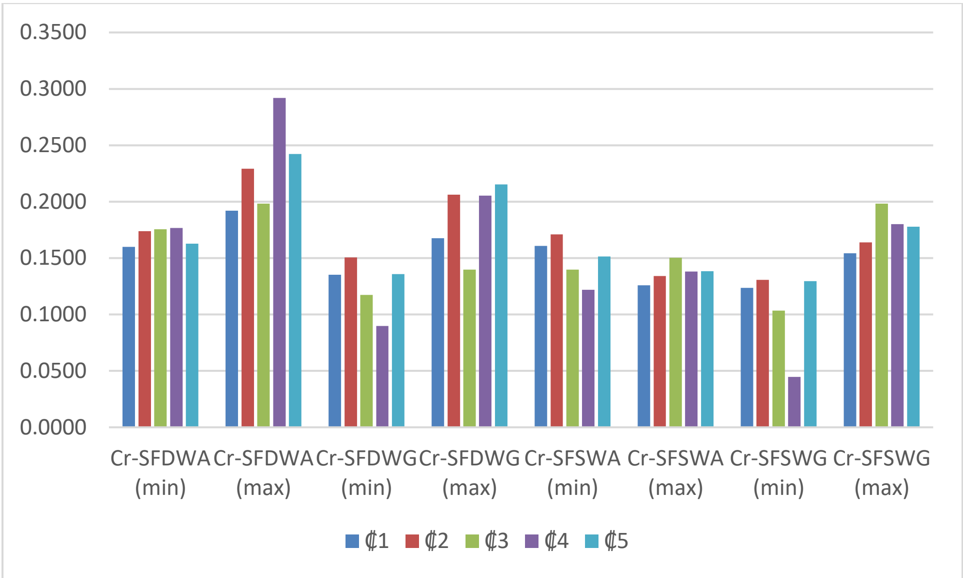

Figure 2 illustrates the results of previous methodologies.

In conclusion, while the foundational work of Ashraf et al. [10] provided critical insights into decision-making under uncertainty, the Cr-SF AOs developed in this study mark a significant leap forward by overcoming the limitations. These operators offer enhanced consistency, adaptability, and robustness, making them the preferred choice for complex and uncertain DM environments. Future research could explore the application of these findings in other domains, further validating the effectiveness of the Cr-SF approach.

7. Conclusion

In this thesis, we have laid a comprehensive foundation for the application of Cr-SFS with the Dombi TN and TCNs. We specifically focused on developing and utilizing four distinct aggregation operators: Cr-SFDWA, Cr-SFDWG, Cr-SFDOWA, and Cr-SFDOWG. These operators were carefully crafted to address the complexities and uncertainties inherent in MADM processes, particularly within the domain of intelligent water sewage system selection. Our research fills a critical gap in the existing literature by extending the utility of SFS into the realm of Cr-SFS, thereby providing a more robust and flexible decision-making framework.

Through a detailed and practical case study, we demonstrated the real-world application of Cr-SFDWA and Cr-SFDWG operators in enhancing water sewage systems with advanced AI technologies. This case study illustrated the theoretical underpinnings of these operators and showcased their practical utility in solving complex MADM problems. The integration of AI into water sewage systems is a growing field, and our work provides a novel approach by leveraging Cr-SFS-based aggregation operators. These operators enable decision-makers to account for multiple, often conflicting, attributes, thus leading to more informed and reliable decisions. A key aspect of our study was the thorough analysis of the behavior of these operators under various parameter adjustments. By varying parameters, such as the weights assigned to different attributes and the sensitivity of the Dombi TN and TCN frameworks, we were able to explore the robustness and adaptability of our proposed operators. This analysis is crucial, as it provides insights into how these operators can be fine-tuned to suit different decision-making contexts, particularly in environments characterized by high uncertainty and complexity.

Author contributions: I am the sole author and contributed to all aspects of this article, including conception, methodology, writing, and revision.

Data availability: The data will be available on reasonable request from the author.

Conflicts of Interest: The author declare there is no conflict of interest.

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