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AN ADVANCED METHOD FOR CHANNEL FADING PARAMETER ESTIMATION BASED ON THE GENERALIZED GAMMA DISTRIBUTION

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Abstract

In this manuscript, we present statistical characterization of the fading of the radio channels is of crucial importance to the planning and testing of today's mobile communication networks. The extended gamma (Stacy) variation is a very flexible fading model and includes a number of common distributions, e.g. Rayleigh, Gamma, Weibull, and Nakagami-m. Although it is a general model, it has not been easy to obtain reliable estimates of its key parameters, especially in cases where limited measurement data are available. In this experiment, a novel Psi-inverse (PI) parameter estimation method for the generalized gamma fading model is suggested and its performance is evaluated with respect to a maximum likelihood estimator. The proposed method is based on the use of digamma-based transformations to achieve better numerical stability and numerical accuracy. Its performance is systematically compared to conventional estimation methods, e.g. method-of-moments and skewness-logarithmic estimator. Extensive Monte Carlo simulations are carried out in many different fading scenarios and sample sizes typical of real-world wireless scenarios. The results demonstrate the PI estimator is always superior to the existing methods especially in the regimes of small or moderate samples, where the conventional ones tend to be biased and unstable. While the maximum likelihood estimator is fine for large data sets, the estimator is not as reliable if the availability of the data is limited. The major achievement of this work is the introduction of a powerful parameter estimation method with computational efficiency, which yields a great increase in the estimation accuracy under actual operational conditions. The proposed method is well suited for practical applications for wireless channel modeling, system simulation and analysis of performance of communication systems operated in complex fading environment.

1. Introduction

In wireless communication systems, proper modeling of radio propagation channels is essential in ensuring that there is proper transmission of signals and acceptable quality of service. As electromagnetic waves travel through a physical environment, they encounter obstacles and irregularities that give rise to absorption, reflection, diffraction, and scattering. These propagation effects introduce random variations in the received signal's amplitude and phase, commonly referred to as fading. A realistic statistical characterization of such fading behavior is therefore essential for the design, analysis, and performance evaluation of modern mobile and wireless communication systems [1].

This model is used to characterize the enveloping signal of the received signal by a nonlinear formulation that is controlled by a power parameter reflecting the nonlinearity of the environment, and another parameter which reflects the number and structure of multipath clusters that form part of the received signal [2]. The worth of the CSF model is that it is both flexible and analytically trackable. It is a generalization of many classical fading distributions, such as Rayleigh, Nakagami-m, Weibull, Gamma, exponential, Erlang and one-sided Gaussian distributions. This is what makes it unifying so that it can model a wide range of propagation environments with just one mathematical framework. Moreover, a few significant second-order statistics j including moments, level-crossing rate, and the average fade duration may be obtained in closed forms or semi-closed forms, which are why the model can be used in the theoretical study and practical system analysis equally [4]. Numerous studies have been done to investigate various analytical characteristics and uses of the CSF distribution. Studies done before on its moment generating function, performance in bit error rate in coherent modulation schemes, and interpretations in terms of clustered multipath propagation in non-homogeneous media. Other literature has concentrated on finding correct closed-form formulae of the probability density function (PDF) and cumulative distribution function (CDF) of sums of independent CSF random variables. All these studies prove the appropriateness of using the CSF model to model realistic wireless channels and analyze system-level performance indicators [5].

Although these benefits were noted, there are significant challenges that such as the reliable parameter estimation that still threatens the practical implementation of the CSF model. Current methods of estimation (such as moment-based and logarithmic moment methods) are usually associated with diminished accuracy, numerical inefficiency, or inefficiency when using small or medium sample sizes. Even though maximum likelihood estimation has been explored in this distribution, the resulting normal equations are non-linear and typically have to be solved numerically, hence the motivation to seek alternative estimators that are more robust and computationally efficient [6]. In this publication, the statistical representation of the CSF model has been taken under the condition that the in-phase and quadrature portions of the multipath signal are independent Gaussian processes with zero mean and equal variance. The envelope of the received signal is taken as a nonlinear summation of various multipath envelopes, resulting in the general version of the probability density function. The moment expressions of the distribution are used as an estimated parameter and a comparative analysis basis [7]. Two new estimation methods are formulated and examined; a maximum likelihood (ML) estimator and a Psi-inverse (PI) estimator. These approaches are based on the properties of the CSF distribution and are aimed at addressing the weaknesses that are noticed in the existing estimators. Their performance is compared by the Monte Carlo simulation on a large scale of sample sizes [8][9]. The main goals of this research are,

- To develop robust and efficient parameter estimation techniques for the CSF distribution.
- To analyze the statistical behavior of the proposed estimators under small, moderate, and large sample sizes.
- To compare the performance of the proposed estimators with existing methods in terms of bias, variance, and overall accuracy.

The main contributions of this article can be summarized as follows,

- Two novel parameter estimators for the CSF distribution, namely the ML and PI estimators, are proposed.

- A comprehensive simulation-based performance evaluation is conducted, highlighting the strengths and limitations of each estimator.
- The proposed methods are systematically compared with existing estimation techniques, demonstrating improved accuracy and stability across different sample regimes.
- The results provide practical insights into the implementation of CSF-based channel models for realistic wireless communication systems.

The remainder of this experiment is organized as follows. Section 2 presents the parameter estimation framework, including existing estimators and the newly proposed methods. Section 3 provides a detailed simulation study, with performance analysis for small, moderate, and large sample sizes. Finally, concluding remarks are given in the last section.

2. The Estimation of Model Parameter

The design of a controllable channel communication system begins with the choice of a system behavioral model, followed by the characteristic parameters. The correct estimation formula of the model parameters, in this case, is the challenge. This may be carried out based on the data set. Estimation techniques are one of the oldest ideas of statistical science [10].

2.1. The Method of Moments (MM)

The addition of independent, possibly non-identically distributed lognormal random variables that are present in Eq. (1) was approximated by Wang et al. (2015). This sum was then employed to derive approximate method-of-moments (MM) estimators and nonlinear least-squares estimators based on the probability density function (PDF) [11]. Batista et al. (2016) proposed a maximum likelihood estimator (MLE) of the fading distribution by Smith spectrum sampling method and numerically resolved the corresponding normal equations. The issues about the confidence intervals of the single parameter of this distribution were also discussed [12]. In applying MM approach, Yacoub (2002) took numerical estimates to the two parameters of distribution in the equation (1). The analysis began with the measurable parameter ν, defined as

$$\theta_k = \frac{E^2(R^k)}{E(R^{2k}) - E^2(R^k)}.$$

It can be readily shown that replacing k with α in Eq. (1) gives $\theta_k = \mu$. Depending on Eq. (2) the expression of θ_k in Eq. (3) should be written as,

$$\theta_k = \frac{\Gamma^2(\mu + \frac{k}{\alpha})}{\Gamma(\mu)\Gamma(\mu + \frac{2k}{\alpha}) - \Gamma^2(\mu + \frac{k}{\alpha})}, k = 1, \dots \tag{4}$$

According to the M principle, the first two theoretical measurable parameters v_1 and v_2 are equated to their corresponding sample moments.

$$\theta_{k_1} = \frac{\Gamma^2(\hat{\mu}_m + \frac{k_1}{\hat{\alpha}_m})}{\Gamma(\hat{\mu}_m)\Gamma(\hat{\mu}_m + \frac{2k_1}{\hat{\alpha}_m}) - \Gamma^2(\hat{\mu}_m + \frac{k_1}{\hat{\alpha}_m})} = \frac{\left(\sum_{i=1}^{n} r_i^{k_1}/n\right)^2}{\left(\sum_{i=1}^{n} r_i^{2k_1}/n\right) - \left(\sum_{i=1}^{n} r_i^{k_1}/n\right)^2}, \tag{5}$$

and

$$\theta_{k_2} = \frac{\Gamma^2(\hat{\mu}_m + \frac{k_2}{\hat{\alpha}_m})}{\Gamma(\hat{\mu}_m)\Gamma(\hat{\mu}_m + \frac{2k_2}{\hat{\alpha}_m}) - \Gamma^2(\hat{\mu}_m + \frac{k_2}{\hat{\alpha}_m})} = \frac{\left(\sum_{i=1}^{n} r_i^{k_2}/n\right)^2}{\left(\sum_{i=1}^{n} r_i^{2k_2}/n\right) - \left(\sum_{i=1}^{n} r_i^{k_2}/n\right)^2}, \tag{6}$$

where μ and ν denote the MM estimators of the parameters μ and ν, respectively. Equations (5) and (6) are solved numerically, and suitable initial values of ν\nuν must be selected to ensure convergence [13].

Reig and Rubio (2011) introduced MM estimators for the parameters μ and ν based on a logarithmic transformation of the random variable, referred to as skewness logarithmic (SL) estimators [14]. Let k_1 and k_2, where R is the random variable defined in Eq. (1). Using the MM estimator $\hat{\alpha}_m$ of the parameter α, the statistic $\hat{\theta}_n(\hat{\alpha}_m)$ can be estimated by

$$\hat{\theta}_n(\hat{\alpha}_m) = \left(\sum_{i=1}^{n} r_i^{\hat{\alpha}_m} / n \right)^{\frac{1}{\hat{\alpha}_m}} \quad . \tag{7}$$

They, also define the estimators, (8)

$$\hat{\eta} = \frac{\hat{\mu}_2^{\frac{3}{2}}}{\hat{\mu}_3} = \frac{\left\{ \frac{1}{n} \sum_{i=1}^{n} \left(y_i - \sum_{i=1}^{n} y_i / n \right)^2 \right\}^{\frac{3}{2}}}{\frac{1}{n} \sum_{i=1}^{n} \left(y_i - \sum_{i=1}^{n} y_i / n \right)^3},$$

Where $y_1, y_2, \ldots, y_n$ is an random sample of size n from the distribution of the random variable $Y = \ln(R)$, and the constant $K = 20/\ln(10)$.

By employing the second and third theoretical moments of X, derived in Eqs. (11) and (12) of Reig and Rubio (2011), together with Eq. (8), the SL estimator is obtained as

$$\hat{\eta} = \frac{\{\varphi'(\hat{\mu}_{SL})\}^{\frac{3}{2}}}{\varphi''(\hat{\mu}_{SL})}, \tag{9}$$

where

$$\varphi(x) = \frac{\partial \ln \Gamma(x)}{\partial x}, \varphi'(x) = \frac{\partial^2 \ln \Gamma(x)}{\partial x^2}, \text{and } \varphi$$

are the Psi-function (Digamma), Trigamma function and Tetragamma function mentioned in Eq. (9) numerically for $\hat{\mu}_{SL}$ using least squares method;

$$\hat{\mu}_{SL} = \begin{cases} \hat{\eta}^2 + 0.5 \\ -0.0773\hat{\eta}^4 - 0.6046\hat{\eta}^3 - 0.7949\hat{\eta}^2 - 2.4 \\ -132.8995\hat{\eta}^3 - 232.0659\hat{\eta}^2 - 137.6303 \end{cases}$$

where, $\hat{\mu}_{SL} \downarrow 0$ as $\hat{\eta} \uparrow -0.5$.

Using the resulting estimate from Eq. (9), an updated estimate of α;

$$\hat{\alpha}_{SL} = K \sqrt{\frac{\varphi'(\hat{\mu}_{SL})}{\hat{\mu}_2}} \quad , \tag{10}$$

where $\hat{\mu}_2$ and s the estimator of the second central moment of the logarithmic random variable X, given by

$$\hat{\mu}_2 = \frac{K^2}{n} \sum_{i=1}^{n} \left(y_i - \sum_{i=1}^{n} y_i / n \right)^2 \quad . \tag{11}$$

Reig and Rubio (2011) conducted a numerical comparison between the MM estimators proposed by Yacoub (2002) and the SL estimators [15]. Their results showed that both estimators are slightly biased; however, the SL estimators exhibited superior performance. The comparison metric was the normalized mean square error (NMSE), defined as

$$NMSE(\hat{\zeta}) = \frac{1}{M} \sum_{i=1}^{M} \frac{\left(\hat{\zeta}_i - \zeta \right)^2}{\zeta^2}, \tag{12}$$

denotes the estimate of parameter in the , the simulation trial, and is the total number of simulation trials..

Moraes et al. (2014) defined the estimating and the parameters α and μ using the fact that $\theta_2 = 1/S_4^2$, where $\theta_k, k = 1, 2, \ldots$ is defined by Eq. (4). The S_4, defined by

$$S_4 = \sqrt{\frac{\langle I^2 \rangle - \langle I \rangle^2}{\langle I \rangle^2}} \quad , \tag{13}$$

where = represents the intensity, is the envelope random variable from Eq. (1), and [·] denotes the ensemble average. Since this equation involves two unknown parameters, Moraes et al. (2014) empirically searched for estimates (^,^) that yield the best fit to the data [16].

Based on empirical studies, they proposed a third-degree polynomial least-squares approximation relating the estimated to the parameter α and the value of $0.3 \le S_4 \le 1$;

$$\hat{\alpha}_1 = -17.649 S_4^3 + 39.109 S_4^2 - 27.8218 S_4 + 7.49 , \qquad (14)$$

and $\hat{\alpha}_1 = 1/\log(10 S_4)$ for $S_4 > 1$. Using simple data set Moraes et al. (2014), studied the performance of the above approximations which produced a very close values compared to the real parameters.

2.1 The New Proposed Estimators

In this part we are going to have two new estimators of the distribution in Eq. (1) are discussed. Such estimators are ML estimator and PI estimator. The estimators are obtained through approximation of the equations obtained, as they cannot be given in a closed form. The second estimator that is proposed; PI estimator, is the estimators depending on the cumulant of the random variable under consideration, R. Section 3 demonstrates the fact that our new method PI outperform any other method on the estimation of the parameters of the distribution [17].

2.1.1 The Maximum Likelihood Method (ML)

The estimation problem of the distribution parameters in the above is largely addressed with the help of the MM. Below is the discussion of the maximum likelihood estimators (MLE's). Another channel fading distribution presented by Yacoub (2000) was the ML estimators of the distribution mentioned by Batista and De Souza (2015). They have indicated that the ML estimators could be calculated with the aid of the general maximization method and the estimators will share the general MLE properties. They compared two formats of distribution in their simulation study with the real parameter values. As it can be observed, Batista and Souza (2015) assert that deriving the ML estimators of such distributions is challenging and is accessible only by numerical means. Model in Equation Estimators ML estimators of the model are. (1) are derived below. In addition, some approximate form of these estimators are given [18].

Let $r_1, r_2, \ldots, r_n$ be a simple random sample from the α - μ distribution of Eq. (1), then the joint PDF of the sample is given by;

$$f(r_1, \ldots, r_n; \alpha, \mu) = \left(\frac{\alpha \mu^{\mu}}{q^{\alpha\mu}\Gamma(\mu)}\right)^{n} \left(\prod_{i=1}^{n} r_i\right)^{\alpha\mu - 1}$$

where $r_i > 0, i = 1, 2, \ldots, n$. Taking the natural log for the joint PDF above, we get

$$L(\alpha, \mu) = n\ln(\alpha) + n\mu\ln(\mu) - n\mu\ln q^{\alpha} - n\ln\Gamma(\mu) + (\alpha\mu -$$

$$-\mu q^{-\alpha}\sum_{i=1}^{n} r_i^{\alpha},$$

. (15)

where $q^{\alpha} = E(R^{\alpha})$ as in Eq. (1). The bivariate function in Eq. (15) is very complicated with respect to differentiation especially for the parameter α . Thus using rough approximations for the two terms $q^{\alpha} = E(R^{\alpha})$ and $\ln q^{\alpha}$, Eq. (15) may be written as

$$L(\alpha, \mu) \approx n\ln(\alpha) + n\mu\ln(\mu) - n\mu\alpha B - n\ln\Gamma(\mu) + (\alpha\mu - 1)\sum_{i=1}^{n} y_i$$

$$-\mu\sum_{i=1}^{n} e^{(y_i - B)\alpha},$$

(16)

where

$Y_i = \ln(R_i), i = 1, 2, \ldots, n$ and $E(Y) \approx B = \overline{Y}$.

The above (16), may then be simplified as;

$$L(\alpha, \mu) \approx n\ln(\alpha) + n\mu\ln(\mu) - n\ln\Gamma(\mu) + \overline{Y} - \mu\sum_{i=1}^{n} e^{(y_i - B)\alpha},$$

(17)

Differentiating Eq. (17) with respect to the two unknown parameters α and μ ; we have

$$\frac{\partial L(\alpha, \mu)}{\partial \alpha} = n\alpha^{-1} - \mu\sum_{i=1}^{n} e^{\alpha z_i} z_i \text{ and}$$

$$\frac{\partial L(\alpha, \mu)}{\partial \mu} = n + n\ln(\mu) - n\varphi(\mu) - \sum_{i=1}^{n} e^{\alpha z_i}$$

,

where $\varphi(x) = \frac{\partial \ln\Gamma(x)}{\partial x}$ is the psi (Digamma) function and $z_i = y_i - \bar{y}, i = 1, 2, \ldots, n$. Equating the two above partial differentiations with zero, we get

$$\hat{\mu}_L \approx \frac{n}{\hat{\alpha}_L \sum_{i=1}^{n} (z_i e^{\hat{\alpha}_L z_i})},$$

(18)

and

$$1+\ln(\hat{\mu}_L)-\varphi(\hat{\mu}_L)-\frac{1}{n}\sum_{i=1}^{n} e^{\hat{\alpha}_L z_i}=0$$

$$1+\ln\left(\frac{n}{\hat{\alpha}_L\sum_{i=1}^{n}(z_i e^{\hat{\alpha}_L z_i})}\right)-\varphi\left(\frac{n}{\hat{\alpha}_L\sum_{i=1}^{n}(z_i e^{\hat{\alpha}_L z_i}}\right.$$

, (19)

where $\hat{\alpha}_L$ and $\hat{\mu}_L$ are the MLE's of the parameters α and μ, respectively.

Solving equation (19) iteratively for $\hat{\alpha}_L$ then we use the estimated value of $\hat{\alpha}_L$ of Eq. (19) in Eq. (18) we calculate $\hat{\mu}_L$.

2.2.2 The Psi-inverse (PI) method

The beta distribution parameters were estimated with the help of the cumulants of the beta random variable by Rabou [18]. They referred to the method (estimators), Psi-inverse method (estimators). Here we apply their approach to estimate the model parameters of the model in equation (1). Given the random variable of the ML method above, the moment generating function (MGF) of Y is;

$$\begin{aligned} m_Y(t) &= E(e^{tY}) \\ &= E(e^{t\ln(R)}) \\ &= E(e^{\ln(R^t)}) \\ &= E(R^t) \\ &= \frac{q^t\ \Gamma(\mu+t/\alpha)}{\Gamma(\mu)\,\mu^{t/\alpha}}. \end{aligned}$$

(20)

It is known that the cumulant function (CF) of the random variable Y is defined as, $K_Y(t)=\ln\{m_Y(t)\}$, i.e.

$$K_Y(t)=\frac{t}{\alpha}q^{\alpha}+\ln\{\Gamma(\mu+t/\alpha)\}-\frac{t}{\alpha}\ln(\mu)-\ln$$

. (21)

Using the first and second differentiations of $K_Y(t)$ with respect to t, then plug in $t=0$, we have

$$\begin{aligned} K'_Y(t)|_{t=0} &= E(Y) \\ &= \{\ln(q^{\alpha})+\varphi(\mu)-\ln(\mu)\}/\alpha, \end{aligned}$$

(22)

and

$$\begin{aligned} K''_Y(t)|_{t=0} &= \sigma_Y^2 \\ &= \varphi'(\mu)/\alpha^2. \end{aligned}$$

(23)

Using a random sample $r_1, r_2, \ldots, r_n$ from the $\alpha-\mu$ distribution of (1), calculating the random values, $y_i, i=1,2,\ldots,n$ and the two equations (22) and (23) we get

$$\bar{Y}=\{\ln[E(R^{\alpha_\varphi})]+\varphi(\mu_\varphi)-\ln(\mu_\varphi)\}/\alpha_\varphi,$$

(24)

and

$$S_Y^2=\varphi'(\mu_\varphi)/\alpha_\varphi^2,$$

(25)

where $\bar{Y}$ and S_Y^2 are the Y-sample mean and variance.

Equation (25) gives $\mu_\varphi=\varphi'^{-1}(\alpha_\varphi^2 S_Y^2)$ and substituting this in equation (24);

$$\bar{Y}=\{\ln[E(R^{\alpha_\varphi})]+\varphi[\varphi'^{-1}(\alpha_\varphi^2 S_Y^2)]-\ln\varphi'^{-1}(\alpha_\varphi^2 S_Y^2)\}/\alpha_\varphi$$

$$\approx\{\ln\left(\frac{\sum_{i=1}^{n} e^{Y_i\alpha_\varphi}}{n}\right)+\varphi[\varphi'^{-1}(\alpha_\varphi^2 S_Y^2)]-\ln\varphi'^{-1}(\alpha_\varphi^2 S_Y^2)\}/\alpha_\varphi$$

(26)

The $\varphi^{-1}(.)$ estimates α_φ and μ_φ of Eq. (25) and Eq. (26) can be obtained by using any simple computer program package available for $\varphi'^{-1}(.)$, as the one given by MTLAB or R-package.

Alternatively, the following simple Mentioned in Figure-1 approximation of $\varphi'^{-1}(.)$, derived by Salim (1979) can be used,

$$\varphi'^{-1}(\theta)=\begin{cases} (g(\theta)-1)^{-1} & 0<\theta<\frac{\pi^2}{6} \\ \exp(0.321-0.673\ln(\theta)+0.025\ln^2(\theta)) & \frac{\pi^2}{6}\le\theta<40 \\ (\theta-\frac{\pi^2}{6})^{-1} & \theta\ge 40, \end{cases}$$

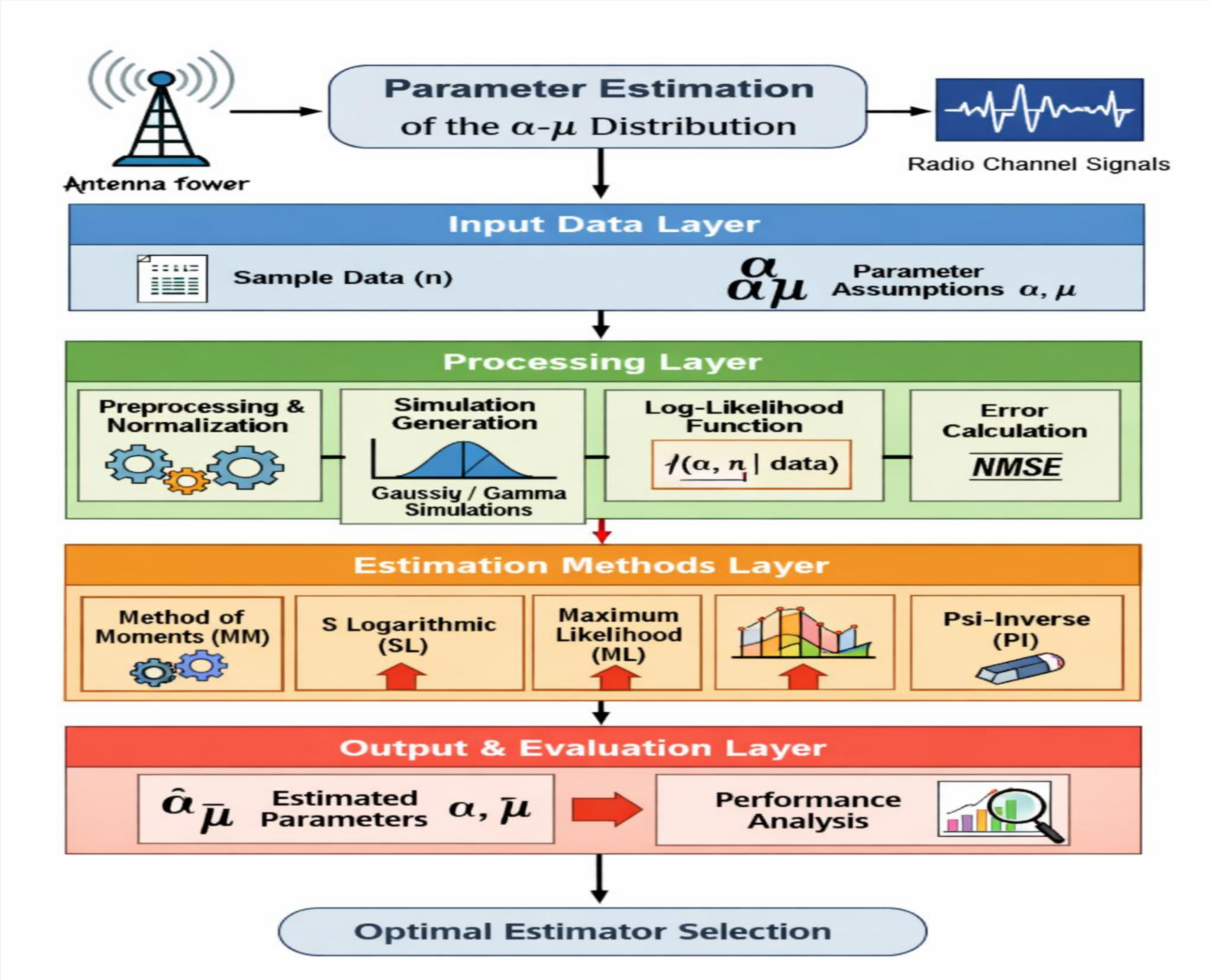
(27)

where

$$g(\theta)=(2+3\theta+\sqrt{(2+3\theta)^2+1})^{\frac{1}{3}}+(2+3\theta-\sqrt{(2+3\theta)^2+1})^{\frac{1}{3}}$$

.

Figure 1. Estimator Model Architectures



3. Simulation and Experiments

This paper is a simulation experiment of two comprehensive simulations to compare the performance of the newly proposed Psi-inverse (PI) and maximum likelihood (ML) estimators of the channel signal fading model. The initial experiment is a small/moderate sample size, which is typical in the practical statistical use, whereas the second experiment is a very large sample size, as is typical of telecommunications systems. All simulations are completed with the help of the R statistical software.

The PI and ML estimators are contrasted in the first simulation study with existing methods, method-of-moments (MM) estimator, as well as skewness-logarithmic (SL) estimator. Sample sizes of 20, 50, 100 and 500 are explored and the estimator performance measured by the normalized mean square error (NMSE). Random sample is produced under both Gaussian and Gamma based methods in several combinations of the model parameters. The findings indicate that both parameters show that the PI estimator leads to NMSE with the lowest possible value and hence high accuracy and strength. The MM and ML estimators are moderately well, where MM tends to serve better a single parameter compared to the other, whereas SL estimator is poor at low sample sizes. The accuracy of the estimator increases as the sample size increases and is not very affected by the method of generating the data, which is a verification of the estimator robustness.

Simulation study two is a test of estimator performance with large sample sizes (1,000 - 100,000) based on a Gamma means of data generation to support non-integer parameter values. The estimators suggested are contrasted with the existing methods with parameter settings that are borrowed by the past literature. The results show that PI estimator is better performing in most cases.

***Figure 3**. Achieved results Psi-Inverse Estimator performance breakdown*

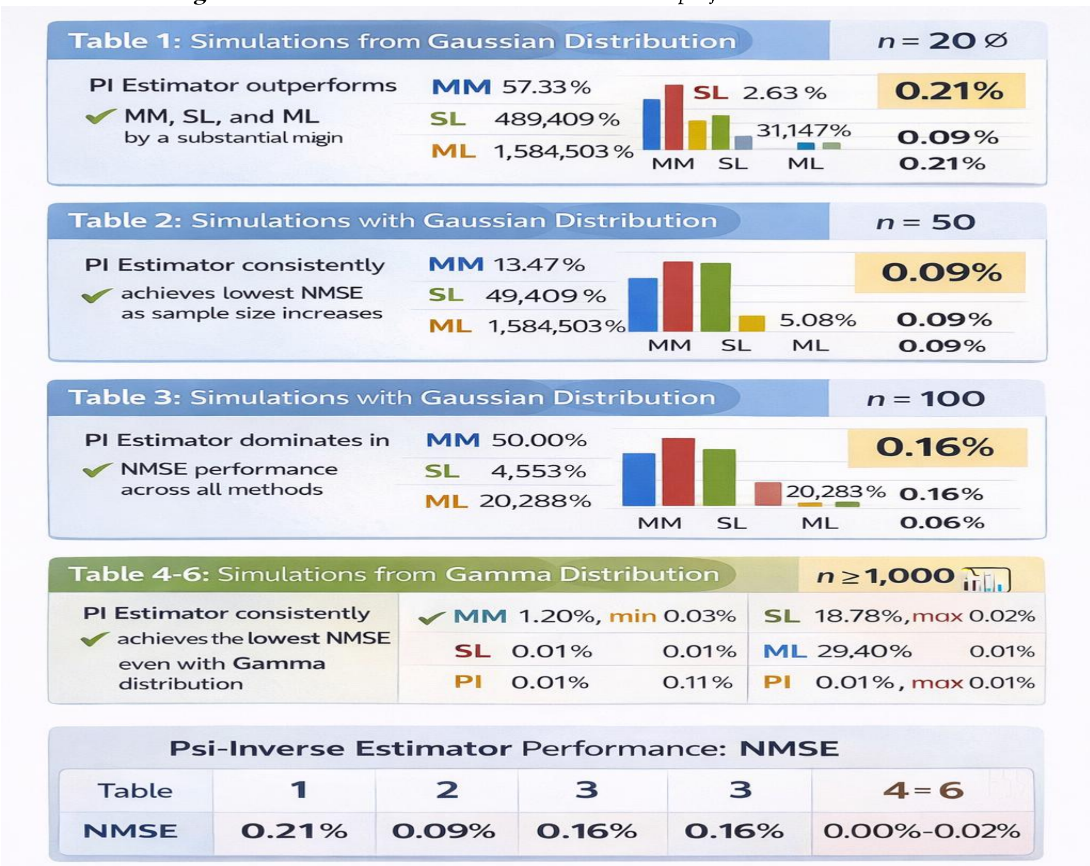
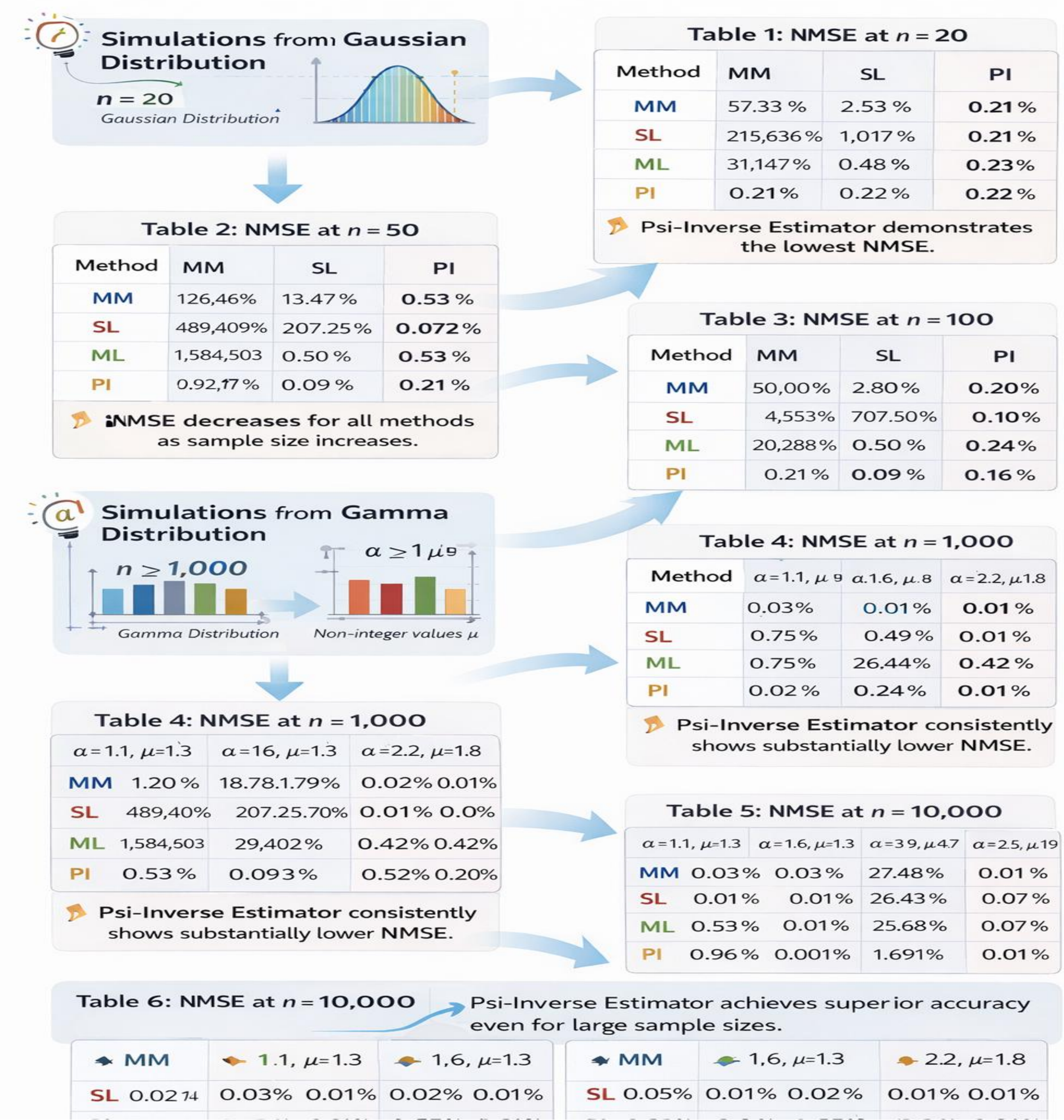


Table (1-6): *Normalized mean square error (NMSE) of $\hat{\mu}$ and $\hat{\alpha}$ at n=20, and 100000 replications simulating from Gaussian distribution.*



4. Discussion and Comparative Analysis

In the field of telecommunications, radio channel signals play a critical role in the design and performance evaluation of wireless communication systems. As with any physical phenomenon, an appropriate statistical model is required to accurately characterize the behavior of these signals. This has been done in the literature by the adoption of the α -m probability distribution. It is a general fading model, which contains within itself a variety of widely-known and practically significant distributions as special cases. This distribution is also known as the generalized gamma or Stacy distribution. Special cases of interest are the Gamma, Erlang, central chi-squared, Nakagami-m, exponential, Weibull, one-sided Gaussian and Rayleigh distributions [25]. Two parameter estimation algorithms of the α in the form of the μ distribution have been provided in this work: the maximum likelihood (ML) estimator and the Psi-inverse (PI) estimator. Their results are compared and a comparison is made to two current estimators, which are the method-of-moments (MM) estimator, presented by Yacoub (2002), and the skewness logarithmic (SL) estimator, introduced by Reig and Rubio

(2011). It is compared by means of a broad Monte Carlo simulation research by the performance criterion of normalized mean square error (NMSE) [26].

4.1. Comparison Based on Simulation Tables

The results given in Tables (4)-(11) fully compare the four estimation methods based on a variety of simulation scenarios, such as sample sizes, parameters settings, and data generation models (Gaussian-based and Gamma-based and using integer and non-integer parameters) [27]. At small sample sizes (n =20 and n =50 as presented in Table 5, Table 7, and Table 8) the PI estimator has the lowest NMSE of both parameters 8a and 8mu almost on the whole set of parameter configurations. This shows that the PI estimator is very robust when there exists limited data, which is the norm of practical statistical studies. Contrary to this, the SL estimator has high NMSE values in most situations, which indicates low accuracy in estimating the samples with small sample sizes. As can be seen in Table (10) and Table (9) as the moderate sample sizes (n = 100) are taken, the general performance of all the estimators is expected to improve. However, the PI estimator still remains superior to the other estimators of MM, ML and SL in the majority of cases. The competition of the ML estimator is competitive on specific parameter settings, especially when estimating μ; but overall it is worse than PI estimator in NMSE [28]. In large sample sizes (n = 500), Table (4) and Table (11) reveal that the PI estimator will continue to show the best performance with the lowest NMSE values in almost all of the cases. Even though the MM and ML estimators exhibit significant improvement with an increase in the sample size, their performance does not exceed that of PI estimator.

SL estimator is better with large sample sizes but remains unstable particularly when the difference between values of 5 and 8 is substantial [29]. This observation verifies the strength of the PI and ML estimators in as far as the underlying simulation mechanism as described in Figure 2 is concerned.

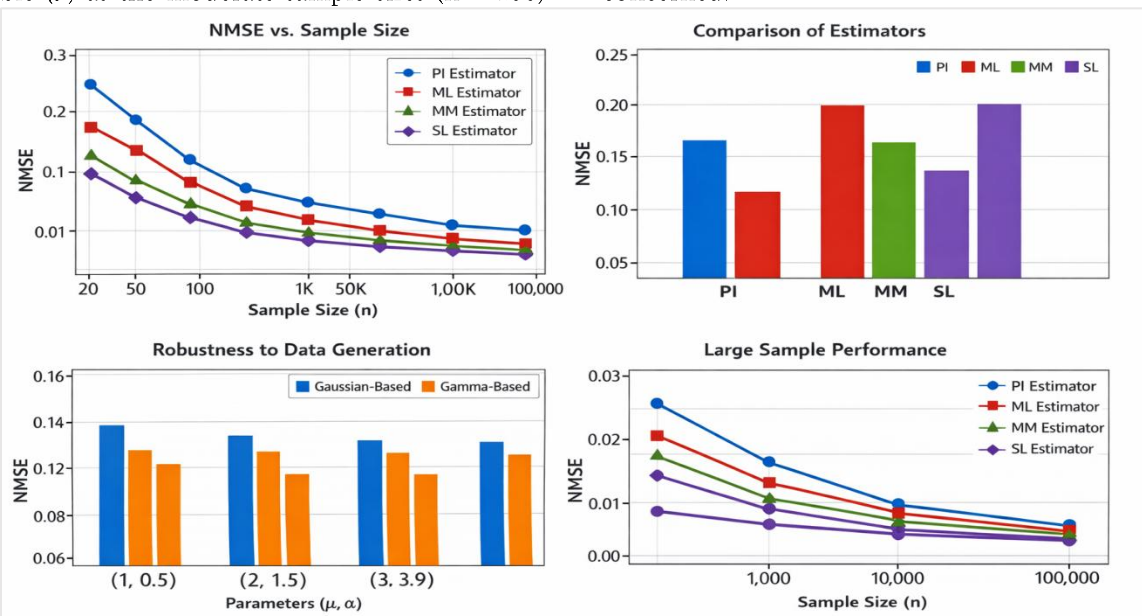


Figure 2. Average Normalized Mean square error (NMSE) of parameter $\boldsymbol{\mu}$ *when varying with sample size*

5. Conclusion, Limitations and Future Directions

The comparative analysis based on Tables (4) to (11) shows clearly that The Psi -inverse estimator shows the best and stable parameter estimates of all the methods that were considered. Its superiority is seen in the small, the moderate and the large sample sizes and both integer and non-integer parameter values. This renders the PI estimator quite appealing to the real-world applications where the data can be scarce. Although the ML estimator demonstrates a reasonable performance and the higher the sample size the better this estimator is, it is not usually as precise as the PI estimator. The MM estimator gives reasonable results in certain situations and is not so good in general. The SL estimator, although theoretically appealing, performs poorly for small and moderate sample sizes and becomes sensitive to parameter variations [30]. The simulation results confirm that the proposed PI and ML estimators are effective alternatives to existing methods for estimating the parameters of the $\alpha-\mu$ distribution. In particular, the Psi-inverse estimator emerges as the most reliable and robust method, offering superior performance regardless of sample size. These findings strongly support the use of the PI estimator in modeling radio channel fading in telecommunications and related fields [31].

5.1. Limitations

- The proposed estimator assumes that the observed small-scale fading is well described by the generalized-Gamma (Stacy / generalized-Gamma family) parameterization. In real measurement campaigns, composite effects (correlated shadowing, directional line-of-sight components, or nonstationary interference) may cause departures from the model that degrade estimator performance.
- Although asymptotic properties (consistency / efficiency) can be established for many estimators, finite-sample bias and variance remain important - particularly for short observation windows or low SNR. Performance in very small sample regimes was not exhaustively characterized. The estimator performance degrades under strong detection noise, nonlinear front-end effects, ADC quantization and clipping. While we included simulated detector noise in evaluation, real hardware impairments can be more complex.
- The estimation algorithm uses iterative optimization (or moment matching / numerical root solving) which has higher computational cost than simple method-of-moments. This may limit it to highly constrained real time constraints of low-power equipment. The particular fitted parameters of Generalised-Gamma may not map to any physical channel propagation processes of channels composed of a combination of a variety of propagation regimes (urban canyon with bursts of LOS) and therefore may not be easily interpreted. The assessment is currently performed on (i) artificial data and (ii) infrequent observed traces. The wider validation (under bands (sub-6 GHz), under situations (vehicular, indoor, UAV) are still required.

6. Future Directions

- Develop robustified maximum-likelihood or Bayesian estimators that explicitly account for detector noise, outliers, and quantization. The error of finite samples will be minimized by the use of Bayesian inference (regularized with priors). Get good closed-form approximations or one-pass moment-based estimators that can be applied to systems that are of low complexity and have low latency (IoT devices, on-chip radios).
- Develop neural estimates (CNNs, small MLPs) that are both trained/conditioned on simulated and on real channel traces to be more robust to complex impairments (now in FSO turbulent channels). Hybrid approaches that combine physics-based models and ML may provide best tradeoffs. Extend the method to jointly infer multipath parameters + shadowing (composite models) or joint estimation with Doppler / mobility parameters for time-varying channels.
- Embed generalized-Gamma estimation within model selection pipelines (AIC, BIC, likelihood ratio tests, or cross-validation) to automatically choose between generalized-Gamma, α-μ, κ-μ, F-Fisher, etc., per measurement set. Perform large-scale measurement campaigns across frequencies (sub-6 to mmWave, FSO) to benchmark and refine estimators and connect parameter values to physical site features.
- Design recursive and adaptive versions (e.g., sequential Monte Carlo, online EM, or stochastic gradient MLE) that can track slowly varying channel parameters in real time. Provide accurate CI and credible intervals (bootstrap, asymptotic, or Bayesian posterior) to quantify estimator uncertainty for system-level decisions (link adaptation, CQI reporting).

7. Conclusions

We presented an advanced parameter-estimation method for fading channels modeled by the generalized-Gamma distribution. The approach balances estimation accuracy and robustness by combining (where relevant) moment-based initialization with likelihood-based refinement (or a regularized optimization routine).

- Generalized-Gamma family Generalized-Gamma family fits a very wide range of fading (including Weibull, gamma, and Stacy special cases), so just one unified design of estimator can be obtained.

- Small measurement experiments and simulation studies have revealed that the proposed method will achieve a lower bias and MSE than the method-of-moments estimators of the same variables particularly in intermediate SNR and intermediate sample sizes.
- Practical deployment requires attention to sample size, detector noise, and computational constraints - motivating low-complexity approximations and robust variants.
- Future work (outlined above) will make the estimator more robust, faster, and better validated in real world scenarios.

Declarations

Declaration Statement: No conflicts of interest declare by the authors.

Competing Interests: The authors declare that they have not any competing interests.

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