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Rigid Image Registration Using L2-Norm Minimization via Coarse Search Approach

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Abstract
Image registration is the process of aligning two or more images of the same scene captured under different conditions. It is widely used in applications such as medical imaging, computer vision, and remote sensing. In this study, we present a rigid image registration approach based on the minimization of an L2-norm objective function. The proposed method employs a coarse search algorithm to explore the parameter space of rigid transformations, including rotation, scaling, and translation. The objective function is defined as the squared difference between the transformed source image and the target image, which is minimized to achieve optimal alignment. The approach is evaluated on both synthetic data (where the target image is generated from the source) and non-synthetic data. The results demonstrate that the coarse search method is capable of achieving accurate registration in cases where the parameter space is adequately sampled. However, its performance depends on the resolution of the search grid. This work highlights the effectiveness of L2-norm-based objective functions in rigid image registration and demonstrates that coarse search can serve as a simple yet reliable optimization strategy.

1. INTRODUCTION

Image registration, the method of superimposing two or more photographs of the same item taken at dissimilar times from dissimilar places using dissimilar sensors. Image registration align two images geometrically [1-6, 8]. It has been used widely in several fields like medical imaging [7] use for diagnosis of disease. Due to different imaging condition, the difference between the images are introduced. In this paper, rigid image registration using an L2-norm objective function via a coarse search algorithm is presented. In image registration, a pair of images is considered: one as the source image and the other as the target image. These two images are defined on a domain Ω. Image registration aims to quantify and minimize the mismatch between the source and the target image. The images considered in this study are grayscale images with pixel intensity values ranging between 0 and.

$$E_{I_1,I_2}(\varphi) = \int_{\Omega} \|I_1 \circ \varphi^{-1}(x_1,x_2) - I_2(x_1,x_2)\|^2 dx_1 dx_2, \quad \forall (x_1,x_2) \in \Omega.$$

In image registration the goal function minimization is considered to reduce the mismatching and to make the images perfectly align as that called perfect registration.

2. Literature Review

example. In 1992. Picture enlistment is generally utilized in far off detecting clinical imaging, PC vision and so on as a general rule, its applications can be partitioned into four fundamental gatherings as indicated by the way of the picture procurement.

Various perspectives (multi view examination): Images of similar scenes are taken from distinguishable perspectives. Its purpose is to obtain a bigger two dimension or three dimension illustration of the filtered scene.

Various occasions (multi worldly investigation): Pictures of similar locations are captured on various occasions. It is often obtained under normal circumstances and in some cases under

One of the medica imaging came from the work of On Growth and Form [9], which is written by evolutionary biologist D'Arcy Thompson, he gave some examples of smooth deformation of a grid overlaid on one type of image, suggesting morphing to another type of image. Comparison of human skull with apes and relationship between several species of animals are given as an various conditions. It is about finding and evaluating changes in the scene that appear during continuous image capture.

Various sensors (multi modal examination): Pictures of similar scenes are taken from various sensors. This is used to combine data from several source streams to create a dynamic and comprehensive scene representation.

Scene to display enrollment: The pictures of the location and the sample of the location are registered. The version can be a PC illustration of the location, such as a GIS map, a digital elevation model (DEM), another scene using a comparator (another patient), or a "normal" example. It's about limiting and or viewing the images captured in the scene model.

3. MOTIVATION

Darcy Wentworth Thompson's original work "Growth and Form" [10] was first published 100 years ago.In a chapter of the book named "On the Theory of Transformation", he found that creatures closely related in metamorphosis space show relatively gentle transformations between their appearances in many situations. Suggested that the variant of may be mathematically "simple". This book has been used for 15 years as the rationale for the study of diffeomorphism in images of various objects. This topic is currently very well studied, especially with regard to medical image enrollment.

This new research effort is mathematically fascinating and seems to have a wide range of uses, but it is not the subject of this treatise. Thompson's main treatise, as mentioned above, was that closely related interspecific transformations should be easy. [11] suggests that this notion of simplicity can be seen as an example of low-dimensional grouping in current languages. Infinite diffeomorphism groups are not easy, but 4D rigidity groups and 6-dimensional Mobius groups are easy. Thompson's work does not include a discussion of equiangular deformations that may seem strange. However, they are common (followed by another example), and two other studies [12] have addressed similar problems, and a two-dimensional model of biological development based on several examples fits. It suggests that it looks like it is. In these studies, cross-ratio between carefully selected locations in the biological growth diagram was used as evidence. [13].

4. RIGID REGISTRATION

In image registration, One of the most often utilized group is the rigid group [13]. For this group the major indicators are scalings, rotations, and arbitrary translations (in 2-dimensional) rigid group [13]. The transformation (which is now) called the rigid transformation) associated with rigid group is explained below:

2.1 Rotation

Suppose an arbitrary $\begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$. Let $\theta \in [0, 2\pi)$ be the angle of rotation transformation $\boldsymbol{\varphi_1}$ to generate the new point $\begin{pmatrix} x' \\ y' \end{pmatrix} \in \mathbb{R}^2$ as defined by:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \varphi_1 \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

2.2 Scaling

Suppose an arbitrary $\boldsymbol{\lambda} \in \mathbb{R}^2$ is a non-zero number so uniform scaling transformation $\boldsymbol{\varphi_2}$ over $\begin{pmatrix} x \\ y \end{pmatrix}$ is defined below:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \varphi_2 \begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix}$$

2.3 Translation

Suppose $\boldsymbol{t_x}, \boldsymbol{t_y} \in \mathbb{R}$ are the translations in the horizontal and vertical axis respectively. The translation transformation $\boldsymbol{\varphi_3}$ over $\begin{pmatrix} x \\ y \end{pmatrix}$ is defined below:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \varphi_3 \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + t_x \\ y + t_y \end{pmatrix}$$

The rigid transformation is the combination of the all above mentioned transformations. The compact form of the rigid transformation $\boldsymbol{\varphi}$ is defined below:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \varphi \begin{pmatrix} x \\ y \end{pmatrix}$$

$$= (\varphi_3 \circ \varphi_2 \circ \varphi_1) \begin{pmatrix} x \\ y \end{pmatrix}$$

$$= \lambda \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}, \quad \theta \in [0, 2\pi), 0 \neq \lambda \in \mathbb{R} \qquad (2.1)$$

The collection of all such $\boldsymbol{\varphi}$ forms a group under composition, which is known as rigid

Group. (From Equation 2.1) The inverse of a rigid transformation is:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}^{-1} \left\{ \begin{pmatrix} x' \\ y' \end{pmatrix} - \begin{pmatrix} t_x \\ t_y \end{pmatrix} \right\}$$

$$= \frac{1}{\lambda} \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}^{-1} \left\{ \begin{pmatrix} x' \\ y' \end{pmatrix} - \begin{pmatrix} t_x \\ t_y \end{pmatrix} \right\} \qquad (2.2)$$

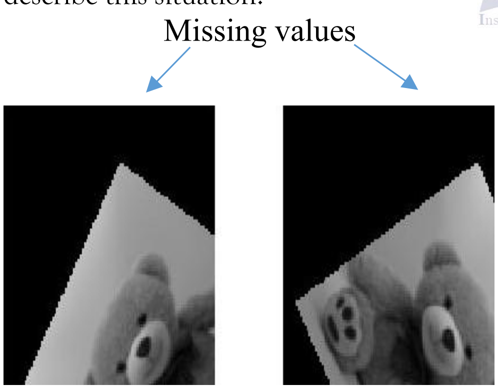
5. IMAGE REGISTRATION (PRACTICALLY)

In this section, we will try to find mathematical and computational methods for accomplishing image registration. This is because the pixel intensity is in the range 0 to 1. Also, the image value that exactly matches the two images is 0.5 for the difference image. A uniform medium gray patch indicates perfect registration. One convenient way to check for discrepancies between images is to subtract or find the difference images. This calculates the contrast between the converted source picture and the target picture and then Rankes it to lie in the range [0, 1]:

$$DI(x,y) = \frac{1}{2}\left(\left(I_1\phi^{-1}(x,y) - I_2(x,y)\right) + 1\right)$$

Lacking values in image registration:

Because the pictures are determined on a limited subset of the plane, transformations will be used to transfer portions of the picture out of the domain and regions of the background into the domain that are now undefined. The first isn't an issue because those regions of the picture are no longer visible, but the other one is because these areas of the picture mirror the places where the backdrop is painted into the image, and so no color instruction is accessible for that one zone. The missing value problem is a term used to describe this situation.



In a pair of images the missing values are shown in black color having value 0.This can be obtained after applying rotational transformation to an image.

Every missing number in our numerical computation is assigned the value 'NaN' (Not-a-Number) by Matlab. As a result, there is no way to compute these positions numerically. The following option can be used to manage with missing values in image registration:

Set c to a random integer that represents all the missing values. For example, in a grayscale image, you can assign the number c in the range [0 1] or set it as the background color of the image. B. Pixel intensity 1 is white and 0 is black.

alternatively, you can ignore the missing values (similar to ignoring mapped values outside the domain) to reduce the norm of differences between overlapping pixels. This means that the total is only calculated for overlapping pixels, not the entire set. However, we have found that this approach does not work on its own, as the objective function can be degraded if the hard mismatch section of the source image is shifted out of the domain.

Algorithm 1: Coarse search approach to minimize Objective Function Equation

Input: I_1 and I_2: source and target images
Output: Warp $\phi{-}1$ and deformed image $I1 \circ \phi{-}1$
For $\theta = -\pi : \pi/3 : \pi$ **do**
  For $\lambda = 0 : 0.1 : 1$ **do**
    For $tx = -0.5 : 0.1 : 0.5$ **do**
      For $ty = -0.5 : 0.1 : 0.5$ **do**

Use bi-linear interpolation to calculate transformed version of the source,

$I_1 \circ \varphi^{-1}(x_{ij}) \forall x_{ij} \in S$

Compute

$d = \|I_1 \circ \phi^{-1}(xij) - I_2(xij)\|^2 \forall x_{ij} \in S$

Here are some examples of perfect registration:

Example 5.1 In this case, the target is generated using the rigid transformation of the source. Parameters used for θ = 1.046, λ = 1, tx = 0.2, ty = 0.3. This example uses Algorithm 1. At last of the optimization, the optimized parameters obtained are θ_opt = 1.046, λ_optm = 1, tx_optm = 0.2, ty_optm = 0.3. Then use the optimization parameters to transform the source. And the result will be displayed in figure 5.1 under the registration we got is a complete registration.

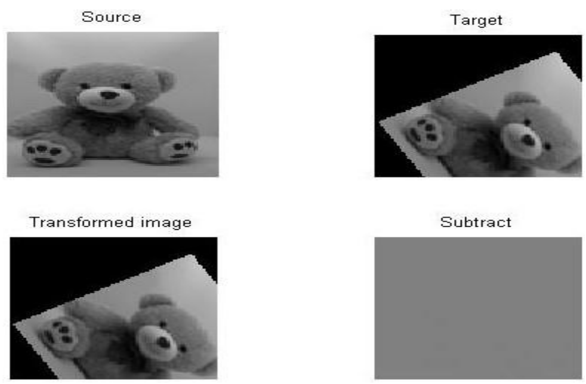


Figure 5.1: In this figure example I_1 and I_2 are overlapping completely and the registration is called perfect registration. Applying algorithm 1 coarse search method.

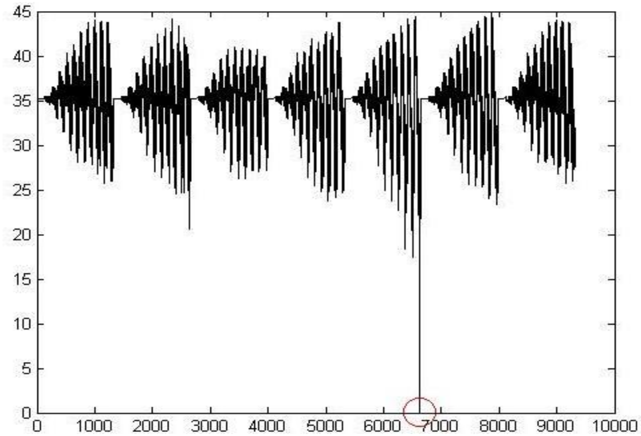


Figure 5.2: In this Coarse search procedure Areas of combined parameter are checked or listed along x-axis and registration fault or errors are plotted along the y-axis. The small circle shows the lowest value obtained by coarse search optimization.

Example 5.2 In this case I_1 and I_2 are overlapping completely and the registration is called perfect registration. The parameters used for θ = 2.093,λ=1,tx =0.1 ,ty=0.3.We now utilize algorithm 1 in this case .At the end of optimization , the optimized parameters that are acquired are θ_{optm} = 2.093,λ_{optm}=1,tx_{optm} =0.1 ,ty_{optm}=0.3 We then use the optimize parameter to transform the source and got perfect registration by minimizing the objective function by coarse search method as shown in figure 5.3:

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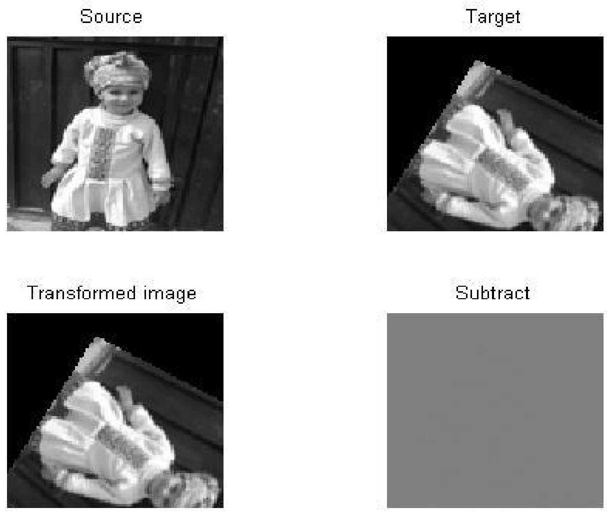
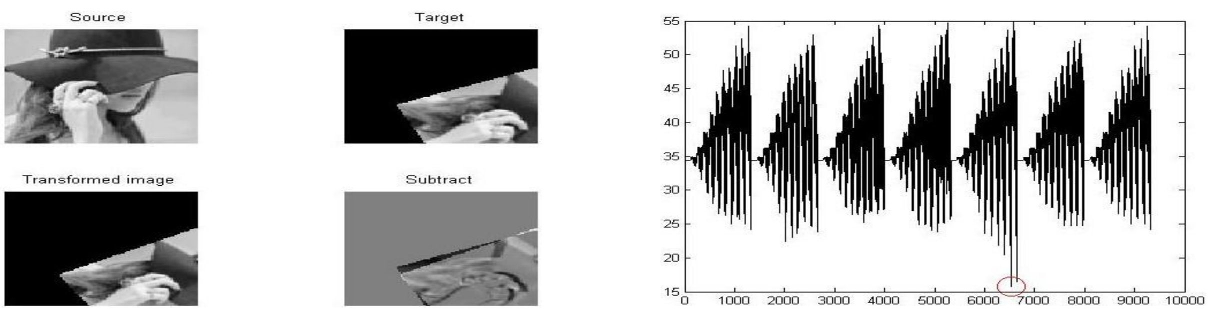


Figure 5.3: In this figure I_1 and I_2 are overlapping completely and the registration is called perfect registration. Applying algorithm 1 coarse search method.

Example 5.3. Figures 5.4 below shows another pair of non-smooth images whose target is generated from a source by rigid transformation. The values of the parameters for this rigid transformation are θ = 1.15, λ = 1, tx = 0.43, and ty = 0.42. Algorithm (1) returns the optimized parameters θopt = 1.0472, λopt = 0.90, txopt = 0.40, and tyopt = 0.40. It is different from the one used to build the target. The problem is that the set of values allowed in the rough search does not include the actual values used to generate the target. Figure 5.4 shows the results of image registration showing valid but not complete registration (perfect registration).



(b)

Figure 5.4: (a): Results of rigid registration for Example 5.3. A good enough, but not perfect, registration is obtained from the coarse search. (b): All outcomes of the objective function for coarse search optimization for Example 2.5. Note that the y-axis only goes down to 20, not to 0 as in the antecedent Examples.

6. CONCLUSION

In this paper, In this work, rigid image registration based on the minimization of an L2-norm objective function has been presented using a coarse search optimization approach. The study focused on aligning source and target images by estimating the optimal transformation parameters through a systematic exploration of the parameter space. The results obtained from multiple examples show that the coarse search method is effective in achieving accurate registration when the true transformation parameters lie within the predefined search domain. In cases involving synthetic data, where the target image is generated from the source, the method successfully achieved perfect registration. However, for more complex or non-synthetic cases, the accuracy of the method depends strongly on the resolution of the search grid. A coarse grid may lead to suboptimal alignment, as the exact transformation parameters may not be included in the search space. This highlights a key limitation of the approach. Despite this limitation, the coarse search method provides a simple, robust, and easy-to-implement framework for rigid image registration. The use of the L2-norm objective function ensures a straightforward formulation based on pixel-wise intensity differences. Future work may focus on improving the efficiency and accuracy of the method by refining the search strategy or incorporating adaptive techniques to better approximate the global minimum of the objective function.

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