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NUMERICAL ANALYSIS OF A PREDATOR–PREY MODEL WITH STAGE STRUCTURE AND SENSITIVITY INVESTIGATION

Rabia[*1], **Sasui**[2], **Dr Ram Chand**[3], **Lubna Naz**[4]

[*1,2,4]*Lecture In Begum Nusrat Bhutto Women University of Sukkur*
[3]*Chairman of Natural Sciences at Begum Nusrat Bhutto Women University of Sukkur*

[*1]rabia@bnbwu.edu.pk, [2]sasui.channar@bnbwu.edu.pk, [3]ram.chand@bnbwu.edu.pk,
[4]lubna.naz@bnbwu.edu.pk

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Corresponding Author: *
Rabia

Abstract

In this paper, we analyze a stage-structured predator-prey model with juvenile and adult classes of both prey and predator populations. The model is formulated as a system of nonlinear ordinary differential equations describing biological processes such as growth, maturation, predation and natural death. The long-term dynamics of the populations is studied by obtaining the equilibrium points of the system and studying their stability. We obtain the basic reproduction number R_0 *by the next generation matrix method as a threshold parameter for persistence in population. The sensitivity analysis of* R_0 *is performed to study the impact of the key parameters. The results show that the prey growth rate has a significant positive effect on* R_0 *and the predation rate has a significant negative effect. Moreover, the numerical simulations are conducted by using Runge-Kutta fourth order (RK_4) method to verify the analytical results. The simulations show the time evolution of all the population classes and confirm the theoretical results about the stability and the behavior of the system. The study discusses the most important parameters governing the dynamics of the predator–prey system.*

1. INTRODUCTION

Predator-prey interactions are an integral part of ecological communities, determining population persistence, ecosystem stability and patterns of biodiversity [1,2]. For a long time, mathematical modeling has been a powerful tool for understanding these interactions, beginning with the seminal Lotka–Volterra model, which was independently introduced by Lotka and Volterra in the early 20th century [3–6]. Since then, many extensions have been developed to include realistic biological complexities such as nonlinear functional responses, environmental heterogeneity, predator interference, and stage-structured population dynamics [7–10].

Particular attention has been given to stage-structured models since many species have distinct juvenile and adult phases with different growth rates, mortality rates and predation responses [11–14]. Ignoring these differences can oversimplify ecosystem dynamics and lead to inaccurate predictions of population stability and resilience. The inclusion of life-stage structure for both prey and predator populations provides a more biologically realistic framework for the analysis of ecological interactions and management strategies.

While traditional predator–prey models have been thoroughly studied for single predator–single prey systems, there are fewer studies on the systems where both predator and prey are divided into juvenile and adult classes [15–18]. The stage structure of a system can have profound effects on its behavior, such as the presence and stability of equilibria, oscillatory dynamics, and long-term persistence. These influences must be understood to predict the responses of ecological systems to environmental changes, harvesting pressures, or behavioral adaptations.

In this paper, we construct and analyze a stage-structured predator-prey model with juvenile and adult classes in both prey and predator populations. We investigate some basic mathematical properties of the system such as positivity, boundedness, equilibrium points and local stability. The basic reproduction number is calculated to determine the persistence or extinction of predator populations. Numerical simulations are carried out to show the dynamic behavior of each population stage and sensitivity analysis is performed to identify key parameters influencing system outcomes.

This work adds to the increasing literature on structured ecological models by providing a detailed theoretical and numerical study of predator-prey interactions with life-stage subdivisions. The results provide insights relevant to ecological management, conservation planning, and to a broader understanding of population dynamics in complex biological systems.

2. Model Formulation

We consider a stage-structured predator–prey system in which both prey and predator populations are divided into juvenile and adult classes. Let

- $x(t)$: juvenile prey population,
- $y(t)$: adult prey population,
- $z(t)$: juvenile predator population,
- $w(t)$: adult predator population.

Thus, the total prey and predator populations are

$$N(t) = x(t) + y(t), \qquad P(t) = z(t) + w(t).$$

The prey population grows through intrinsic reproduction and transitions from juvenile to adult stages, while being reduced by natural mortality and predation by juvenile and adult predators. The predator population increases through consumption of juvenile and adult prey and through stage transition, and decreases via natural mortality and intraspecific competition within each stage.

The system of differential equations describing the dynamics is formulated as follows:

$$\frac{dX}{dt} = rY - b_1X - d_1X - \frac{m_1XW}{k_1 + X},$$

$$\frac{dY}{dt} = b_1X - c_1Y^2 - \frac{m_2YW}{k_2 + k_3Y^2} - d_2Y,$$

$$\frac{dZ}{dt} = \frac{\alpha m_1XW}{k_1 + X} + \frac{\beta m_2YW}{k_2 + k_3Y^2} - nZ - d_3Z - \eta YW,$$

$$\frac{dW}{dt} = nZ - d_4W.$$

All model parameters are non-negative and are interpreted as follows:

Table 1: parameters

Parameter	Description	Value
$\mathbf{r}$	Birth rate of juvenile prey, proportional to existing adult prey.	4.9
$\mathbf{b_1}$	Transition rate from juvenile prey to adult prey.	2.8
$\mathbf{c_1}$	Coefficient of interspecific competition among adult prey.	0.06
$\mathbf{m_1}$	Attack rate of predators on prey.	0.06
$\mathbf{k_1}$	Half saturation constant for prey consumption by predators.	5.2
$\mathbf{d_1}$	Death rate of juvenile prey.	0.5
$\mathbf{d_2}$	Death rate of adult prey.	0.4
$\boldsymbol{\alpha}$	Conversion efficiency from juvenile prey to predator growth.	6.2
$\mathbf{n}$	Transition rate from juvenile predator to adult predator.	5.9
$\mathbf{d_3}$	Death rate of juvenile predator.	1.8
$\mathbf{d_4}$	Death rate of adult predator.	3.4
$\mathbf{m_2}$	Attack rate of predators on prey.	4.9
$\mathbf{k_2}$	Half saturation constant for predator consumption by prey.	1.4
$\mathbf{k_3}$	Half saturation constant for predator consumption by prey.	0.09
$\boldsymbol{\beta}$	Conversion efficiency from adult prey to predator growth.	3.3
$\boldsymbol{\eta}$	Rate of anti-predator behavior of adult prey towards predators.	1.5

This system captures stage-specific growth, predation, and mortality processes, allowing a biologically realistic representation of the interactions between juvenile and adult populations of both prey a and predators.

3. Positivity and boundedness

3.1 Positivity

For a population model to be biologically meaningful, all state variables must remain non-negative for all future time whenever they start from positive initial conditions. In this section, we establish the positivity of solutions of the proposed stage-structured predator–prey model.

Let

$$V(t) = \big(X(t), Y(t), Z(t), W(t)\big)^T$$

denote the state vector of the system, where X(t),Y(t),Z(t)X(t), and W(t) represent the juvenile prey, adult prey, juvenile predator, and adult predator populations, respectively. We consider the positive invariant region

$$\mathbb{R}_+^4 = \{(X, Y, Z, W) \in \mathbb{R}^4 : X, Y, Z, W \geq 0\}$$

The system can be written in vector form as

$$\frac{dV}{dt} = \Theta\big(V(t)\big),$$

where

$$\Theta(V(t)) = \Big(\theta_1\big(V(t)\big), \theta_2\big(V(t)\big), \theta_3\big(V(t)\big), \theta_4\big(V(t)\big)\Big)^T$$

To prove positivity, we examine the behavior of the vector field on the boundary of $\mathbb{R}_+^4$.

On the hyperplane X=0, we obtain

$\theta_1\big(V(t)\big) = rY \geq 0.$

On the hyperplane Y=0,

$\theta_2\big(V(t)\big) = b_1 X \geq 0$.

On the hyperplane Z=0,

$$\theta_3\big(V(t)\big) = \frac{\alpha m_1 XW}{k_1 + X} + \frac{\beta m_2 YW}{k_2 + k_3 Y^2} - \eta YW \geq 0$$

for non-negative state variables and parameters.

On the hyperplane W=0,

$\theta_4(V(t)) = nZ \geq 0$

Thus, on each boundary plane of $\mathbb{R}_+^4$, the corresponding rate function is non-negative. Therefore, the vector field $\Theta(V(t))$ points inward or is tangent to the boundary of the positive orthant.

By Nagumo's theorem, the region $\mathbb{R}_+^4$ is positively invariant under the flow of the system. Hence, for any initial condition

$V(0) \in \mathbb{R}_+^4$,

the solution V(t) satisfies

$X(t), Y(t), Z(t), W(t) \geq 0$for all $t \geq 0$.

Therefore, the proposed stage-structured predator–prey model admits positive solutions, ensuring its biological feasibility.

3.2 Boundedness

Boundedness is a key property for ensuring that population densities remain within biologically feasible limits over time. To establish boundedness for the proposed stage-structured predator–prey model, we consider the weighted total population function

$N(t) = \alpha X(t) + \beta Y(t) + Z(t) + W(t),$

Differentiating $N(t)$ along the trajectories of the system gives:

$$\frac{dN}{dt} = \alpha\frac{dX}{dt} + \beta\frac{dY}{dt} + \frac{dZ}{dt} + \frac{dW}{dt}$$

Substituting the system equations into this derivative, we combine growth, predation, and mortality terms. Introducing the constant

$\xi = \min\{d_1, d_2, d_3, d_4\},$

which represents the minimum natural death rate among all population classes, allows us to form the augmented derivative.

$$\frac{dN}{dt} + \xi N$$

For sufficiently large populations, the mortality and saturation terms dominate the growth components, ensuring that there exists a positive constant M such that

$$\frac{dN}{dt} + \xi N \leq M$$

Solving this differential inequality yields

$$N \leq \frac{M}{\xi} + \left(N(0) - \frac{M}{\xi}\right)e^{-\xi t}, t \geq 0,$$

demonstrating that the total population remains uniformly bounded over time. Consequently, all state variables $X(t), Y(t), Z(t)$, and $W(t)$are bounded within the positive invariant region.

4. Equilibrium points

Equilibrium points of a dynamical system represent states at which the system remains unchanged over time. Mathematically, an equilibrium point is obtained by setting the time derivatives of all state variables equal to zero. In ecological models, equilibrium points correspond to population configurations where species densities remain constant, reflecting long-term steady states of the system.

For the proposed stage-structured predator–prey model, equilibrium points describe population levels of juvenile and adult prey as well as juvenile and adult predators that persist without temporal variation. The equilibrium points are obtained by solving the system

$$\frac{dX}{dt} = \frac{dY}{dt} = \frac{dZ}{dt} = \frac{dw}{dt} = 0$$

4.1 Trivial equilibrium point

The trivial equilibrium point occurs when all population variables vanish, that is,

$E_0 = (0,0,0,0)$

Substituting $X = Y = Z = W = 0$ into the system equations, each differential equation is satisfied identically. Hence, the system admits the trivial equilibrium point, which corresponds to the extinction of all species.

4.2 Predator-free equilibrium point

The predator-free equilibrium corresponds to the absence of predators while prey populations persist. Let

$E_1 = (X, Y, 0, 0)$

Substituting $Z = W = 0$ into the system yields

$rY - (b_1 + d_1)X = 0,$

$b_1X - c_1Y^2 - d_2Y = 0.$

From the first equation,

$$X = \frac{rY}{b_1 + d_1}\ .$$

Substituting this expression into the second equation gives

$$\frac{b_1 r}{b_1 + d_1} Y - c_1Y^2 - d_2Y = 0$$

which can be written as

$$Y = \left(\frac{b_1 r}{b_1 + d_1} - c_1Y - d_2\right) = 0$$

Thus, either $Y = 0$ leading back to the trivial equilibrium, or

$$Y = \frac{1}{c_1}\left(\frac{b_1\, r}{b_1 + d_1} - d_2\right)$$

Substituting this value of Y into the expression for X, we obtain

$$X = \frac{r}{c_1(b_1 + d_1)}\left(\frac{b_1\, r}{b_1 + d_1} - d_2\right)$$

Hence, the predator-free equilibrium point is

$$\left[\frac{r}{c_1(b_1 + d_1)}\left\{\frac{b_1\, r}{b_1 + d_1} - d_2\right\}, \frac{1}{c_1}\left(\frac{b_1\, r}{b_1 + d_1} - d_2\right), 0, 0\right]$$

Provided

$$\frac{b_1\, r}{b_1 + d_1} > d_2$$

4.3 Interior (Coexistence) Equilibrium Point

The interior equilibrium point, also called the coexistence equilibrium, corresponds to a state where all populations coexist with strictly positive densities. Let

$E^* = (X^*, Y^*, Z^*, W^*)$

Setting the right-hand sides of the system equal to zero, we obtain

$$rY = b_1X + d_1X + \frac{m_1XW}{k_1+X},$$
$$b_1X = c_1Y^2 + \frac{m_2YW}{k_2 + k_3Y^2} + d_2Y,$$
$$(n + d_3)Z = \frac{\alpha m_1XW}{k_1 + X} + \frac{\beta m_2YW}{k_2 + k_3Y^2} - \eta YW,$$
$$W = \frac{nZ}{d_4}.$$

From these equations, the interior equilibrium can be expressed implicitly as

$$\left[\frac{1}{b_1}\left\{c_1Y^2 + \frac{m_2YW}{k_2 + k_3Y^2} + d_2Y\right\}, \frac{1}{r}\left\{(b_1 + d_1)X + \frac{m_1XW}{k_1 + X}\right\}, \frac{1}{(n + d_3)}\left\{\frac{\alpha m_1XW}{k_1 + X} + YW\left(\frac{\beta m_2}{k_2 + k_3Y^2} - \eta\right)\right\}, \frac{nZ}{d_4}\right]$$

The existence of the interior equilibrium depends on the positivity of these expressions and appropriate parameter conditions, ensuring coexistence of prey and predator populations at both juvenile and adult stages.

5. Stability analysis

The Jacobian matrix is applied to analyze the local stability of the suggested stage-structured predator–prey model. The equilibrium points are found by putting the system of differential equations to zero. Then, the system is linearized around each equilibrium point and the stability behavior is obtained by the eigenvalues of the corresponding Jacobian matrix.

Let f_1, f_2, f_3, f_4 be the right-hand sides. The Jacobian matrix is:

$$\begin{bmatrix} \frac{\partial f_1}{\partial X} & \frac{\partial f_1}{\partial Y} & \frac{\partial f_1}{\partial Z} & \frac{\partial f_1}{\partial W} \\ \frac{\partial f_2}{\partial X} & \frac{\partial f_2}{\partial Y} & \frac{\partial f_2}{\partial Z} & \frac{\partial f_2}{\partial W} \\ \frac{\partial f_3}{\partial X} & \frac{\partial f_3}{\partial Y} & \frac{\partial f_3}{\partial Z} & \frac{\partial f_3}{\partial W} \\ \frac{\partial f_4}{\partial X} & \frac{\partial f_4}{\partial Y} & \frac{\partial f_4}{\partial Z} & \frac{\partial f_4}{\partial W} \end{bmatrix}$$

5.1 Stability of Trivial Equilibrium $\mathbf{E_0} = (\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0})$

Evaluating the Jacobian at E_0, we obtain:

$$J(E_0) = \begin{bmatrix} -(b_1 + d_1) & r & 0 & 0 \\ b_1 & -\frac{d_2}{k_2} & 0 & 0 \\ 0 & 0 & -(n + d_3) & 0 \\ 0 & 0 & n & -d_4 \end{bmatrix}$$

The characteristic equation reduces to:

$$\{(n + d_3) + \lambda\}\{d_4 + \lambda\}\left\{\lambda^2 + \lambda\left(d_1 + b_1 + \frac{d_2}{{k_2}^2}\right) + \frac{d_2}{{k_2}^2}(d_1 + b_1) - rb_1\right\} = 0$$

Thus, two eigenvalues are:

$\lambda_1 = -n - d_3,$

$\lambda_2 = -d_4,$

which are negative.

The remaining eigenvalues depend on:

$$\frac{d_2}{{k_2}^2}(d_1 + b_1) - rb_1.$$

Hence, the trivial equilibrium E_0 is locally asymptotically stable if:

$$\frac{d_2}{{k_2}^2}(d_1 + b_1) > rb_1,$$

and unstable otherwise.

5.2 Stability of Predator-Free Equilibrium $\mathbf{E_1 = (X, Y, 0, 0)}$

The Jacobian at E_1 takes the form:

$$J(E_1) = \begin{bmatrix} -(b_1 + d_1) & r & 0 & P_1 \\ b_1 & -P_2 & 0 & -P_3 \\ 0 & 0 & -(n + d_3) & -P_4 \\ 0 & 0 & n & -d_4 \end{bmatrix}$$

Where,

$P_1 = \frac{m_1X}{k_1+X}$, $P_2 = 2c_1Y + d_2$, $P_3 = \frac{m_2Y}{k_2+k_3Y^2}$ and $P_4 = \frac{\alpha m_1X}{k_1+X} + \frac{\beta m_2Y}{k_2+k_3Y^2} - \eta Y$

The characteristic equation can be expressed as:

$$[\lambda^2 + (n + d_3 + d_4)\lambda + \{d_4(n + d_3) - nP_4\}][\lambda^2 + P_2(d_1 + b_1)\lambda + P_2(d_1 + b_1) - rb_1\}] = 0$$

The equilibrium E_1 is locally asymptotically stable if:

$0 < P_4 < \frac{d_4(n+d_3)}{n}$, and $P_2(d_1 + b_1) > rb_1$.

5.3 Stability Analysis for the interior equilibrium point $\mathbf{E^* = (X^*, Y^*, Z^*, W^*)}$:

$$J(E^*) = \begin{bmatrix} -Q_1 & r & 0 & -Q_2 \\ b_1 & -Q_3 & 0 & -Q_4 \\ Q_5 & Q_6 & -Q_7 & Q_8 \\ 0 & 0 & n & -d_4 \end{bmatrix}$$

Where,

$Q_1 = -(b_1 + d_1) - \frac{m_1Wk_1}{k_1+X}$, $Q_2 = \frac{m_1X}{k_1+X}$, $Q_3 = 2c_1Y - \frac{m_2W(k_2 - k_3Y^2)}{k_2+k_3Y^2} - d_2$, $Q_4 = \frac{m_2Y}{k_2+k_3Y^2}$, $Q_5 = \frac{\alpha m_1Wk_1}{k_1+X}$, $Q_6 = \frac{\beta m_2W(k_2-k_3Y^2)}{k_2+k_3Y^2} - \eta W$, $Q_7 = (n + d_3)$ and $Q_8 = \frac{\alpha m_1X}{k_1+X} + \frac{\beta m_2Y}{k_2+k_3Y^2} - \eta Y$

The characteristic equation is:

$$\lambda^4 + A\lambda^3 + B\lambda^2 + C\lambda + D = 0$$

Where,

$A = Q_1 + Q_3 + Q_3 + d_4$

$B = Q_1Q_7 + Q_1d_4 + Q_3Q_7 + Q_3d_4 + Q_7d_4 - rb_1$

$C = Q_1Q_3Q_7 + Q_1Q_3d_4 + Q_1Q_7d_4 + Q_7Q_3d_4 - rb_1Q_7 - rb_1d_4 - Q_2Q_6n + Q_2Q_5$

$D = Q_1Q_3Q_7d_4 + Q_8n - Q_4Q_6n - Q_7d_4b_1r - Q_4Q_5rn + Q_1Q_5Q_7 - Q_2Q_6n - Q_2Q_3Q_6n$

By the Routh–Hurwitz criterion, the interior equilibrium E^* is locally asymptotically stable if:

$A > 0, B > 0, C > 0, D > 0, AB > C, ABC > C^2 + A^2D.$

6. Numerical Simulation

The predator-prey model was solved numerically by the fourth-order Runge-Kutta (RK_4) method, which is one of the most widely used methods for integrating non-stiff ordinary differential equations. The first order ODE system is:

Hypothetical values were used (Table 1) owing to the lack of real data for all parameters. The RK_4 scheme was used with a step size h=0.1, which was large enough to reproduce the population oscillations with good accuracy and low computational effort.

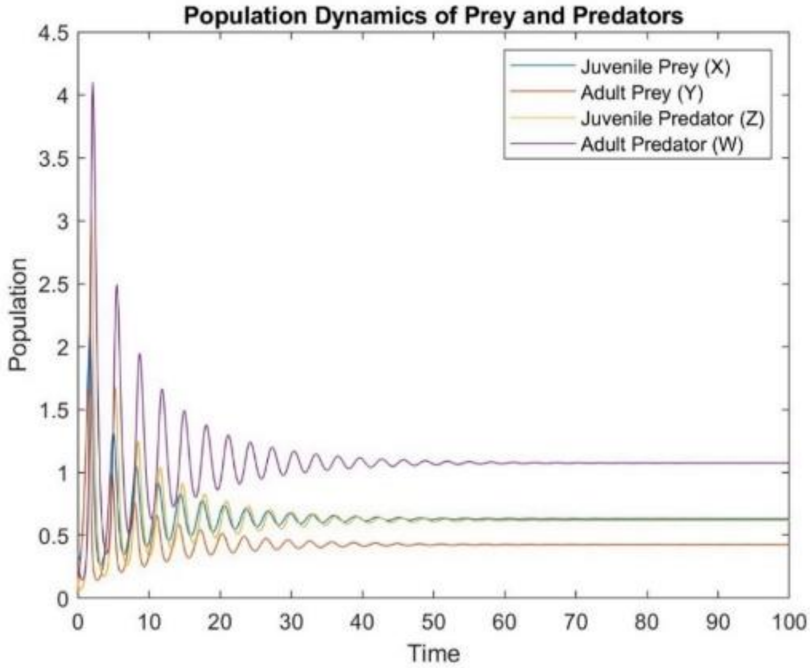

Fig1: Population Dynamics of Prey and Predator Species Over Time by RK_4 scheme, step size h = 0.1

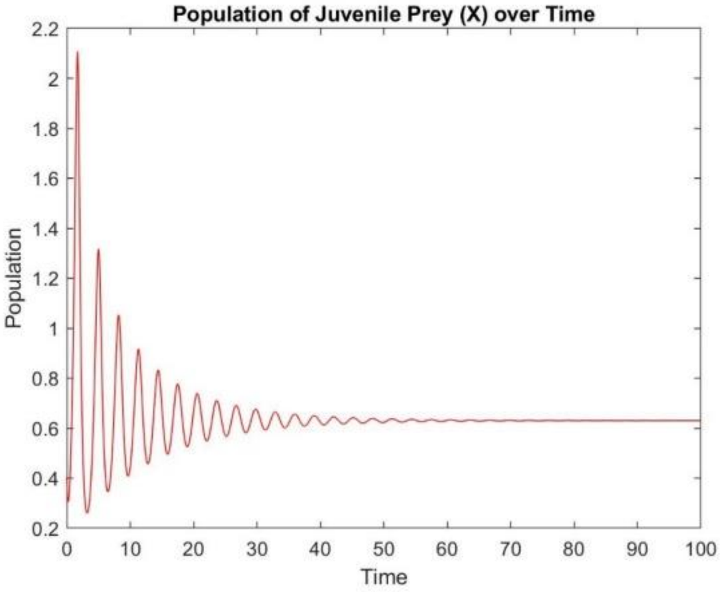

Fig 2: Population Dynamics of Juvenile Prey (X) Over Time

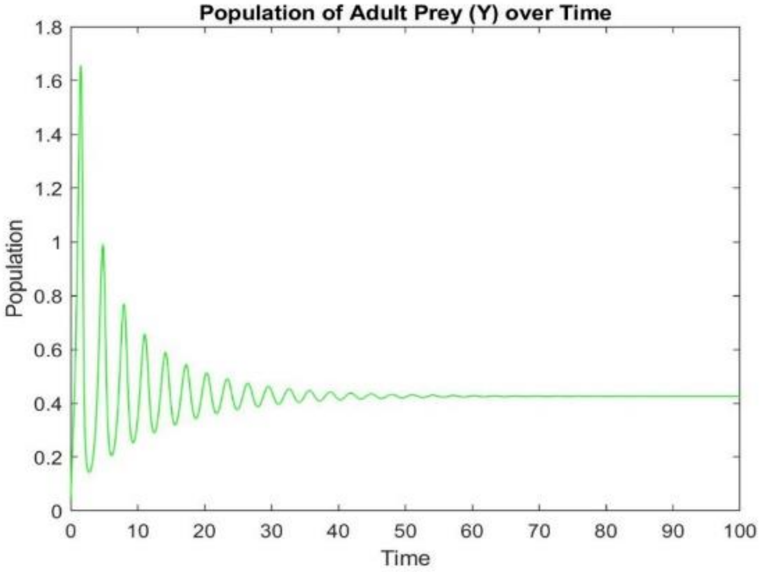

Fig 3: Population Dynamics of Adult Prey (Y) Over Time

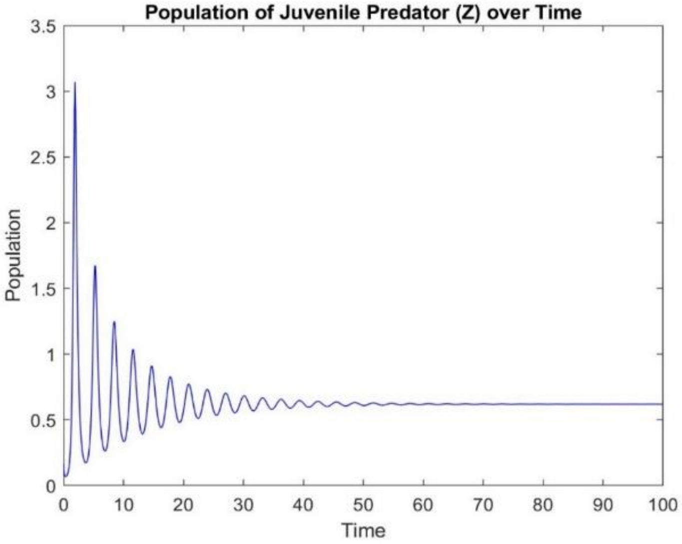

Fig 4: Population Dynamics of Juvenile Predator (Z) Over Time

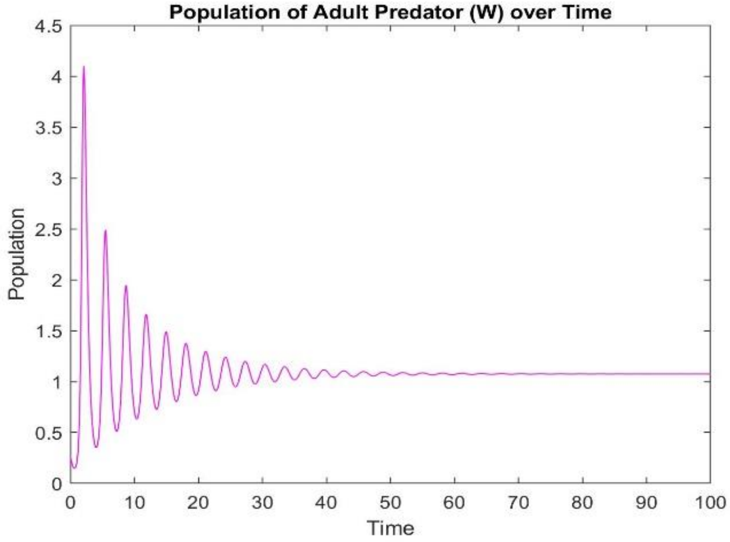

Fig 5: Population Dynamics of Adult Predator (W) Over Time

RK_4 simulation showed oscillatory dynamics of populations through the life stages. We observed coupled oscillations in juvenile and adult prey populations (Figures 1–3) and lower oscillations of juvenile predators compared to adult predators (Figures 4–5). Predation is driven mainly by adult predators, which control prey population dynamics.

The results underscore the importance of stage-specific dynamics in predator-prey systems. The oscillatory behaviors indicate the sensitivity of the system to the ecological parameters, and the importance of numerical simulations in understanding the population stability and ecosystem management.

7. Basic Reproduction Number R_0 in Predator-Prey Model

The basic reproduction number R_0 is a key quantity in the study of population dynamics. In predator-prey systems, R_0 represents the average number of offspring produced by the prey population in the presence of predators and is a measure of whether the predator population can persist.

In the predator-prey model with stage structure, the prey population reproduces according to its birth and predation rate, while the predator population reproduces according to its consumption rate and prey availability. Mathematically, the system is described by:

$$\frac{dX}{dt} = rY - b_1X - d_1X - \frac{m_1XW}{k_1 + X},$$
$$\frac{dY}{dt} = b_1X - c_1Y^2 - \frac{m_2YW}{k_2 + k_3Y^2} - d_2Y,$$
$$\frac{dZ}{dt} = \frac{\alpha m_1XW}{k_1 + X} + \frac{\beta m_2YW}{k_2 + k_3Y^2} - nZ - d_3Z - \eta YW,$$
$$\frac{dW}{dt} = nZ - d_4W.$$

To compute R_0, we apply the next-generation matrix method. Let F denote the matrix of new offspring terms and V denote the matrix of transition and loss terms:

$$\mathcal{F}=\begin{pmatrix} -\frac{m_1 W k_1}{(k_1+X)^2} & 0 & 0 & -\frac{m_1\, X}{k_1+X} \\ 0 & -2c_1 Y-\frac{m_2 W\left(k_2-k_3 Y^2\right)}{(k_2+k_3 Y^2)^2} & 0 & -\frac{m_2 Y}{k_2+k_3 Y^2} \\ \frac{\alpha m_1 W k_1}{(k_1+X)^2} & \frac{\beta m_2 W\left(k_2-k_3 Y^2\right)}{(k_2+k_3 Y^2)^2}-\eta W & 0 & \frac{\alpha m_1 X}{k_1+X}+\frac{\beta m_2 Y}{k_2+k_3 Y^2}-\eta Y \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\mathcal{V}=\begin{pmatrix} b_1+d_1 & -r & 0 & 0 \\ -b_1 & d_2 & 0 & 0 \\ 0 & 0 & n+d_3 & 0 \\ 0 & 0 & -n & +d_4 \end{pmatrix}$$

The next-generation matrix is FV^{-1}, and the largest eigenvalue of this matrix corresponds to R_0. Using the equilibrium point:

$$(X,Y,Z,W)=\left[\frac{r}{c_1(b_1+d_1)}\left\{\frac{b_1\, r}{b_1+d_1}-d_2\right\},\frac{1}{c_1}\left(\frac{b_1\, r}{b_1+d_1}-d_2\right),0{,}0\right]$$

we obtain:

$$R_0=\frac{2(b_1 r-b_1 d_2-d_1 d_2)}{b_1^2(d_2-r)+b_1 d_1 r+d_1^2 d_2-2b_1 d_1 d_2}$$

Substituting the parameter values $b_1 = 2.8, d_1 = 0.5, d_2 = 0.4, and\ r = 4.9$ yields:

$$R_0 = -0.8244 < 1$$

Since R_0 the predator population is too small to control the growth of the prey. As a result, the prey population increases, and the predator population decreases due to a lack of food. This explanation fits with ecological theory which states that when predators are too few relative to prey, prey flourish and predator populations decline.

8. Sensitivity Analysis of the Basic Reproduction Number

To assess the impact of model parameters on the system dynamics, we performed a sensitivity analysis of the basic reproduction number R_0. The normalized forward sensitivity index of R_0 with respect to a parameter P_i is defined as:

$$\Gamma_{P_1}^{R_0}=\frac{dR_0}{dP_i}\times\frac{P_i}{R_0}$$

This index measures the relative change in R_0 due to a relative change in parameter P_i. A positive value of the sensitivity index indicates that an increase in the parameter increases R_0, whereas a negative value indicates that an increase in the parameter decreases R_0

Analytical Expression of R_0

The basic reproduction number is given by:

$$R_0=\frac{2(b_1 r-b_1 d_2-d_1 d_2)}{b_1^2(d_2-r)+b_1 d_1 r+d_1^2 d_2-2b_1 d_1 d_2}$$

Using this expression, sensitivity indices were computed for the parameters $r, b_1, d_1, and\ d_2$.

Table 2: sensitivity indices

Parameter	Sensitivity Index $\Gamma_{P_1}^{R_0}$	Effect on R_0
r	2.9022	Strong positive effect
b_1	-1.4114	Strong negative effect
d_1	0.1743	Weak positive effect
d_2	-0.034	Weak negative effect

Interpretation of Results

The results indicate that the prey growth rate r has the most significant positive influence on R_0. This means that an increase in r leads to a substantial increase in the system's growth potential.

On the other hand, the predation rate b_1 has a strong negative sensitivity index. This implies that increasing b_1 reduces R_0, reflecting the suppressive effect of predation on the prey population.

The parameter d_1 (natural mortality of juvenile prey) shows a small positive sensitivity, indicating a slight increase in R_0 with increasing d_1. In contrast, d_2 (adult prey mortality) has a negative sensitivity index, meaning that increasing d_2 decreases R_0.

Overall, the system is most sensitive to changes in prey growth rate r and predation rate b_1, which act as the primary drivers of the population dynamics.

Graphical Interpretation

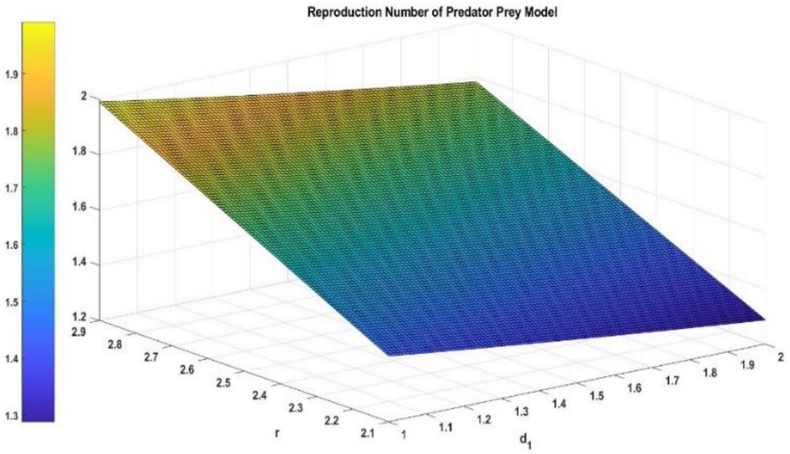

Fig 6: r vs d_1:

The variation of R_0 with respect to selected parameters is illustrated in Figures 6–8:
R_0 increases significantly with increasing r, while only a slight increase is observed with d_1

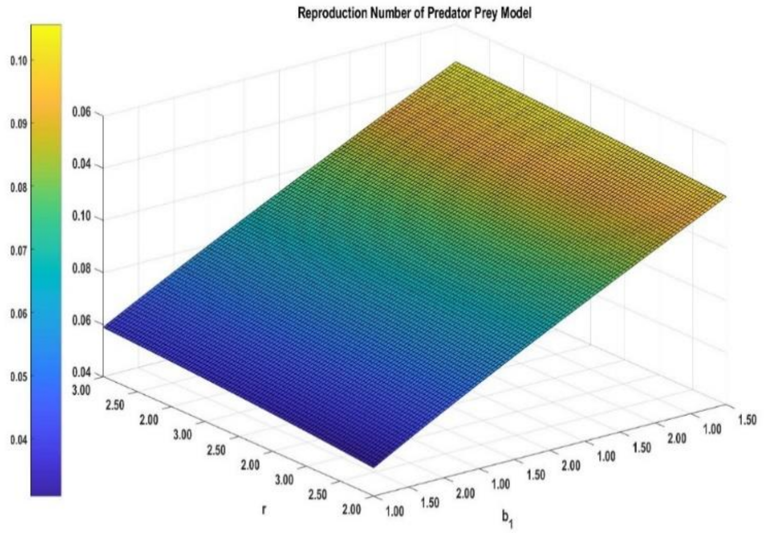

Fig 7: $\boldsymbol{r}$ vs $\boldsymbol{b_1}$

R_0 increases with r but decreases with increasing b_1, confirming the negative sensitivity of b_1.

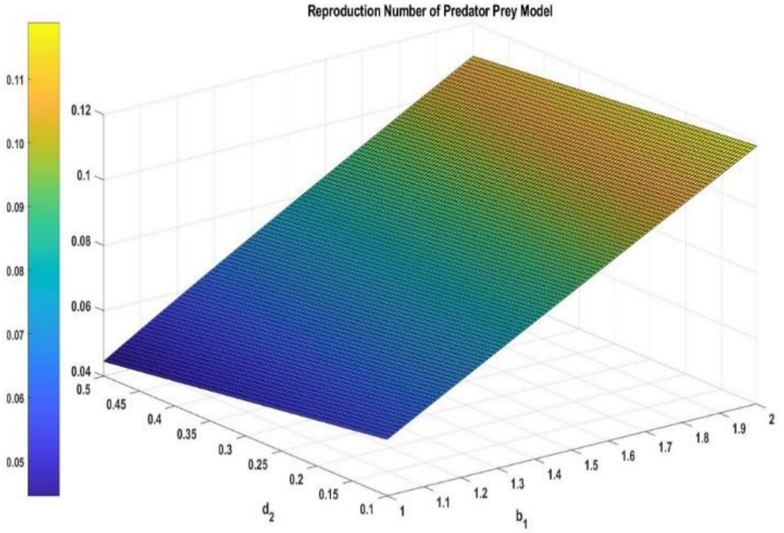

Fig 8: $\boldsymbol{d_2}$ vs $\boldsymbol{d_1}$

R_0 decreases as d_2 increases, whereas d_1 shows a mild positive effect.

9. Conclusion

In this study, a stage-structured predator–prey model with juvenile and adult classes was developed and analyzed using a combination of analytical and numerical techniques. The model captures important ecological interactions including growth, maturation, predation, and mortality, making it a more realistic representation of population dynamics.

The equilibrium points of the system were identified and a stability analysis was carried out to investigate the long-term behavior of the model. The results suggest that system dynamics are very sensitive to parameter values, especially those relating to growth and predation processes.

The basic reproduction number R_0 was obtained by next generation matrix approach. If $R_0<1$, then this means that the predator population is not able to sustain itself at the given parameter values and so the predator population declines and the prey dominates. Sensitivity analysis indicated that the system is most sensitive to the prey growth rate and predation rate, which play a dominant role in the dynamics.

The analytical results were confirmed by numerical simulations using the RK_4 method, which also showed the temporal behavior of all population classes. The simulation results are consistent with the equilibrium and stability analysis.

This study provides useful insights into predator–prey interaction and highlights key parameters affecting population dynamics. These results may be useful for understanding ecological balance and designing strategies for ecosystem management and conservation.

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