OPTION PRICING IN THE LIGHT OF ISLAMIC VISION FOR PAKISTAN STOCK EXCHANGE

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Abstract

It is widely known, The Black-Scholes model is frequently used to establish the behavior of the options trading in the financial market. Throughout this paper we address the issue of Islamic Al-Arboun (the down payment) that is similar to the conventional options trading. We proposed the modified version of the 2-D time-fractional Black-Scholes partial differential c with two assets established on the combination of Finite-Volume Method for unsteady flow and numerical scheme. This work deals with the analytical solution of the European Call option based on financial derivative is so called modified Finite Volume (unsteady) options which numerically solved. Through mathematical analysis it is established that the explicit Finite-Volume scheme is unconditionally stable. After analyzing the conceptual and legal differences between the conventional and Islamic (Al-Arboun) options trading we conclude that Al-Arboun could be the shari'a compliant alternative to the European conventional Call option.

INTRODUCTION

This study plot the work on strategically investing in the stock market through highly analytical and numerical solutions of the famous non-linear P.D.E known as the time-fractional Black-Scholes equation, aligning it with thematic study of sharia compliant to correlate with practically useable for end users, such as industry, banks, and stock markets around the world.

The last decades have seen a rise in financial market volatility. Derivatives, the most frequent tools for hedging financial risks, have shown exponential growth throughout the same time span. Derivatives have not supported in the stabilization of markets or the mitigation of the financial crisis. It's not difficult to understand, given that derivatives are the most common speculative tools. Speculations account for more than 97 percent of derivatives trades, while hedging procedures account for less than 3%. The task at hand is to develop instruments that enable for the control of production risks while avoiding speculation. In the globe, Islamic finance is undergoing a huge transformation. With far more than 250 Financial institutions, its assets were valued at more than 1,400 billion dollars in 2012, with a rate of growth ranging among both 20% and 30% based on the region of operation, which in most cases is substantially greater than regular financial activities.

"Al-Arboun" has the potential to provide a basis for the creation of Shari'a compliant options, which become agreements in which one party purchases the right to acquire specified things from the other at a specific rate and at a specific time from the other party. Al-Arboun" is similar to the Call option in that the seller does not refund the premium (Down payment) if the buyer somehow doesn't exercise the payment method, and the buyer lose the risk premium (Down payment) unless the option is exercised and the agreements is presented..

The options premium will be adjusted in the sale amount while the agreement is confirmed in the case of Al-Arboun. As a result, we can see the feasibility of modeling "Al-Arboun" for modifying the Black-Sholes model to Shari'a rules in order to price the deposit amount. Islamic finance has been gaining popularity in a variety of organizations and institutions, such as banks and stock exchanges, in both Muslim and non-Muslim countries. The most difficult difficulty, however, appears to be meeting the computational and mathematical strategically current forms of transaction and business density, particularly in Pakistan's stock market.

OPTION PRICING IN THE LIGHT OF ISLAMIC VISION:

Option pricing is a rapidly emerging topic of financial mathematics that seeks to solve problems in financial market place pricing. It is a relatively new field that consists of approaches for forecasting asset prices in the financial market. With the help of many solution approaches from applied mathematics, applied statistics, econometrics, software engineering, and computer science, the Black Scholes mathematical model can address complicated financial markets, stock exchange problems. The Black Scholes model is one of the most widely used financial models, and it was the first to anticipate the pricing of the options (put and call) as well as implied volatility. Successful entrepreneurs are typically looking for the biggest reward, which can only be achieved when risk is kept to a minimum and profit is maximized. Because share values are very volatile and unpredictable, the stock exchange is a good illustration of option pricing. The well-known Black-Scholes PDE, but at the other hand, may be used to determine risk-free pricing.

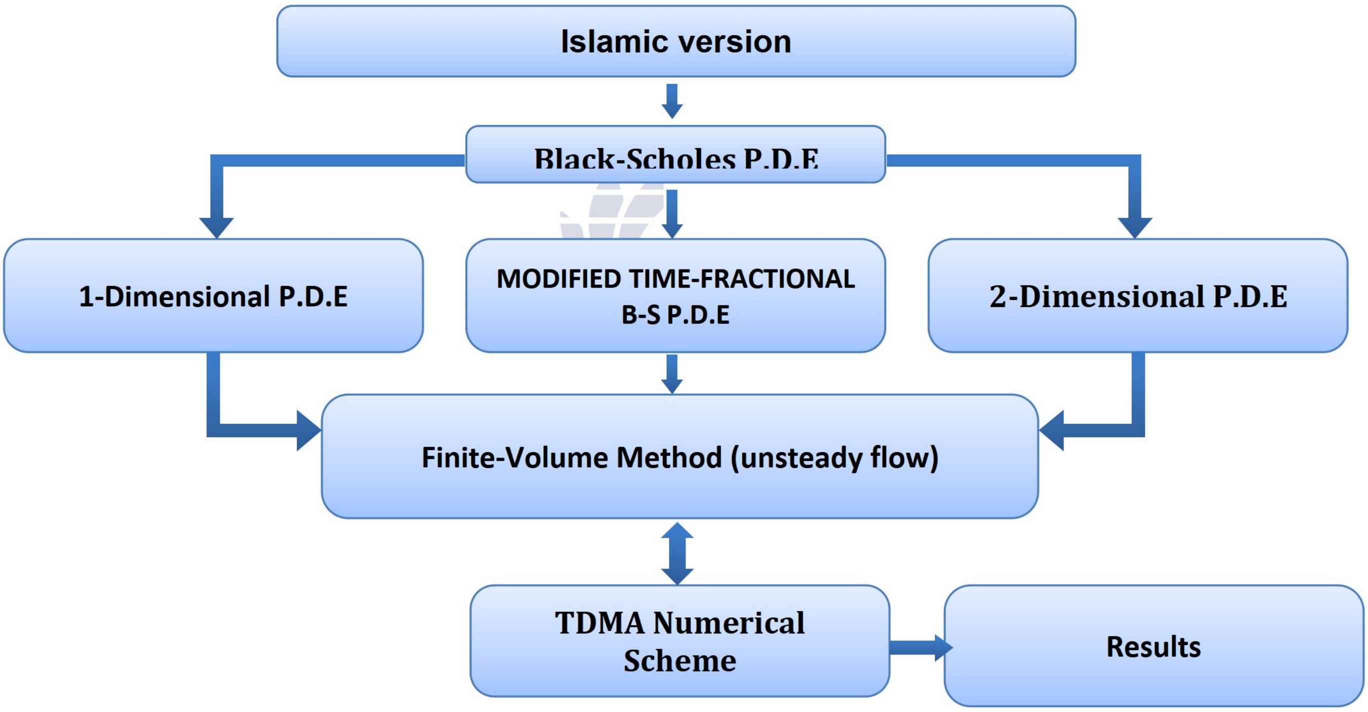
Since the 1980s, Islamic banking has attracted a lot of attention, especially after Islamic banks were able to absorb the shocks of the debt crisis that rocked international banks in 1990. Islamic finance has now been acknowledged globally as a fully-fledged system, and foreign financial institutions have expressed interest in this new business. The International Monetary Fund (IMF) is the most notable example, having published its first report on Islamic banking in 1987. The success that has made Islamic financial institutions essentially immune to the 2008 financial crisis explains the contemporary desire for Islamic banking (Boumediene and Caby 2009). As a result, numerous market participants have viewed Islamic financial solutions as an alternative to traditional investment vehicles. Indeed, throughout the last two decades, this industry has had enormous growth rates, which are currently projected to be between 10% and 15%. (Brack 2007; McKenzie 2011)

Significance of Black-Sholes Model for Stock-Exchange of Pakistan:

- Playing a vital role during political disturbance and un-stable environment in the country, maintaining financial growth of economy in Pakistan.

- Improving Fiscal and monetary policies.
- Globalization of Economy.
- Financial Market system will perfectly beneficial from smoothly functioning stock-Exchange.
- Liquidity and market efficiency on stock Exchange will improve.
- Pronouncing risk factor for stock's exchange can be eliminated by diversification. Investor use option they will stiffer less when fluctuation the stock Exchange.
- Foreign investor will come in Pakistan would be more comfortable to buy Or write the options.
- The launch of BS-Model in Pakistan is the best step for development of human capital. It grows positive Structural changes in Banking-system, insurance policies, encouragement of entrepreneur-ship, reducing the transaction cost.
- The Market participant can easily manage their part of risk exposure in an efficiency way.
- To obtain debt leverage easily.
- Financial institutions can acquire debt without any difficulty for strengthen their business operation smoothly with their design interest rates.
- Providing price and interest rate infrastructure for options which is the clue about future trend of stock-exchange to develop business for buyer and writer.

RESEARCH METHODOLOGY



COMPARISION BETWEEN CONVENSIONAL OPTIONS AND ISLAMIC OPTIONS

What are options: A stock option is a contract between two parties. When you buy a stock option, you are purchasing the right, but not the duty, to buy or sell a stock at a specified price within a specific time frame. If you're selling options, you must comply with the "Shares options allow you to profit from a fluctuation in a company's stock in an intensified or leveraged manner," contract based on the buyer's selection. The profit might come from a rise or fall in the stock's price. Several key terms are important to discussing and understanding how options work

Holders and writers: The option writer sells the option, whereas the option holder buys it.

Exercising options: When the holder has decided to buy or sell the shares they are exercising an option.

Expiration date: The end of the purchase or sale period.

Premiums: For the option, the holder pays a nonrefundable premium to the writer. Its value is determined by the expiry date and the projected volatility of the underlying stock.

Strike or exercise price: The price at which the holder can purchase or sell the shares is known as the exercise price. The option holder (the buyer) can execute the option at any time period before the expiration with American-style options. European-style options, on the other hand, may only be exercised upon that expiration date.

A call option: Means the holder can buy the stock at a specific price during a specific period of time.

A put option: Means the holder can sell the stock at a specific price during a specific period of time. You can buy or sell either type of option.

Islamic options

AL-Arboun (Call option): The Al-Arboun and conventional Call option are contracts in which one party buys the right to purchase specified stock from the other at a defined price on a certain date. The down payment is a non-refundable fee that the buyer must pay when the contract is signed. It is the cost of the contract's optional component. Unlike the premium for a call the down payment is included in the global price rather than being a separate amount.

Financial Derivatives:

It is the contract between two parties, according to this contract: any physical cash commodity can be buy or sold between the parties cash commodities may be oil, petrol, gold , financial instruments like as bond, stocks, interest rate or market index etc.

Classification of Financial Derivatives:

There are 4 types of Financial Derivatives contracts i.e.

1. Forward
2. Future
3. Swaps
4. Options

Forward Contracts:

It is the contract between two parties, one has commitment to buy and other has commitment to sell a specific physical asset or stock at a pre-decided price, on a particular date in future time.

Future Contracts:

Future contract have standardized norms for trading and exchange between two parties to buy and sell the specific physical asset or stock on a particular date on Future time.

Swaps Contracts:

Swap is the type of contract between two parties agrees to exchange a series of cash flow on periodic settlement dates over a certain period of time.

Options or options Derivatives:

An option is a contract in which the owner (holder or buyer) has the right but not the obligation to purchase or sell an underlying asset at a pre-determined price (strike price) on a future date (Expiration Date) in the future (stock at a specific price within a specific time period). Assume a person acquires an option contract to buy 100 shares of State Bank of Pakistan for Rs.250 for a four-month term from the date of the transaction. If the stock price rises, he will exercise the option; if the stock price falls, he will not exercise the option.

Types of options:

1. Call option
2. Put option

Call Option:

A contract that gives the right to buyer (holder of asset/stock) to purchase the principal asset/stock from the seller (writer) for a specific pre-decided price and time in future. It is an option to buy i.e. purchase.

Example: Premium = Rs.4 strike price = 120

Investor will get the profit, when share price is more than Rs.160

Call Option is the amount of money while strike price is below the stroke price

Call Option holder must pay the premium to the writer for the take in an option.

Put Option:

Put option is the contract that gives the right to the buyer (Stock's holder) to sell the asset to the seller (writer) at a pre-decided price on specific date in future. Put Option is the amount of money when strike price is more than stock's price.

Example: Strike price = Rs.120
Premium paid = Rs.40
investor will get the profit when share-price of stock falls below to Rs.80

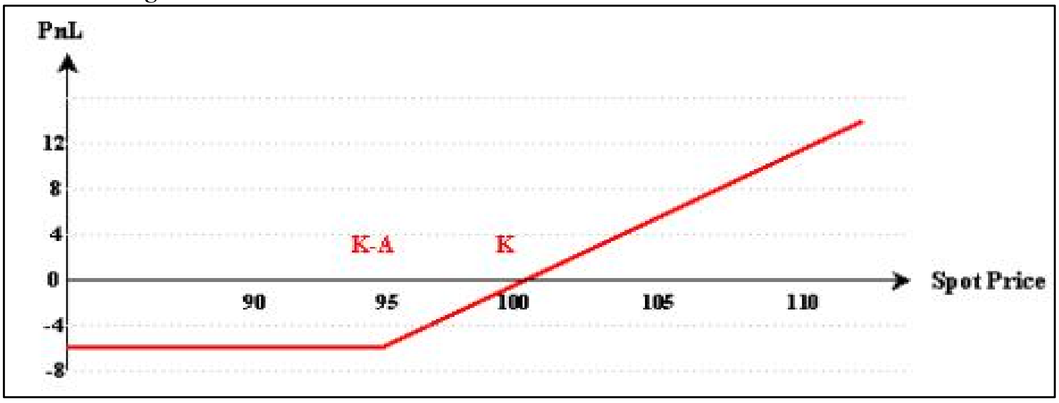
AL-Arboun call option:

The Al-Arboun and conventional Call option are contracts in which one party buys the right to purchases the right to purchase specified stock from the other at a defined price on a certain date. The down payment is a non-refundable fee that the buyer will pay when the contract is signed. It is the cost of the contract's optional component. Unlike the premium for a call the down payment is included in the global price rather than being a separate amount.

Consider S = stock price K= strike price call option price per share = deposit per share

Illustration:

Profit and loss diagram of Al-Arboun



Al-Arboun is a contract in which the parties agree on a $100 global price=K and a $5 deposit. The buyer should cancel the agreement if the spot price is less than the remaining amount (K-A = $95). On the other hand, if a buyer does not cancel the contract, he will only be responsible for the remaining $95 in addition to the $5 deposit.

The profit and loss diagram of Al-Arboun appears to be identical to that of a call from a methodological perspective, the Al-Arboun allows the buyer to fully explain the increase in disbursing just a little amount compared to the global price and restrict downside risk to the deposit amount, much as the call does.

Table: ***P&L of Al-Arboun***

Value of underlying Asset	Profit and Loss of the buyer	Profit and Loss of the seller
S ≤ K – A	-A	+A
K – A ≤ S ≤ K	(S – K) < 0	(K – S) > 0
S > K	(S – K) > 0	(K – S) < 0

Volatility of Stock's Price:

It is the S.D and measure the variation in the rate of stock's Price. If volatility increases then more than one investor has willing to pay for the option. Volatility of stock's act significantly, if volatility of stock's increase than option price for both put and call will be increased vice versa. Volatility measures the uncertainty in the expected future price of asset in future.

Summary of Variables Affecting Call and Put Prices Effect on

Factor	Call Option Price	Put option price
Increase In the price of asset (S)	Increased	Decreased
Increase In Strike price (K)	Decreased	Increased
Increase In asset's Volatility (r)	Increases	Increased
Increase In Expiration time (t)	Increased	Increased
Increase risk free Interest rate (r)	Increased	Decreased

Black Sholes Model:

The B-S model is a financial benchmark, and it was the first mathematical formula to anticipate the market of options (including call and put) as well as implied volatility. We try to incorporate shariah concepts into the fundamental premises of the black Scholes model (two-dimensional). This model employs five primary variables that have a significant impact on the pricing of options. These are the five variables;

a) Stock price
b) Strike price

c) Risk free interest rate
d) Relative Price Volatility of underlying asset
e) Maturity time

Assumptions Of BS-Model:

- Volatility of stock remains constant till to date of expiration.
- No dividend payment on stock during the life of option.
- Returns on underlying stock/asset must follow the normal Distribution.
- Option is European, only be exercised at maturity time T.
- The Market is frictionless, liquidity is perfect, no asymmetry information, trading in continuous time.
- Absence of arbitrage opportunity (there is no way to make a riskless profit)
- Returns earned on investment on stock/asset are known constant risk free interest rate.

Partial Differential Equation Of Black-Scholes Model For Options Pricing:

The below Partial Differential Equations defines the Model for both American styles put option and European style call option price of the stock. These so-called PDE are said to be Black-Sholes Model.

$$\frac{\partial P}{\partial t}+\frac{1}{2}\partial^2 S^2\frac{\partial^2 P}{\partial S^2}+rS\frac{\partial P}{\partial S}-rP=0 \quad \text{------- (i)}$$

Subject to P(S, 0) = Max (K-S, 0)

$$\frac{\partial C}{\partial t}+\frac{1}{2}\partial^2 S^2\frac{\partial^2 C}{\partial S^2}+rS\frac{\partial C}{\partial S}-rC=0 \quad \text{------- (ii)}$$

Subject to *P(S, 0) = Max(S-K, 0)*

Where Models (i) and (ii) based on the following five Financial Inputs:

P = Put option price of stock.
C = Call option price of stock
S = Price of share of stock
∂ = Price volatility or S.D of stock
K = Strike price or Exercise price of stock
t = Time to expiry in years.

Geometric Brownian motion for movement of single stock price and two stock prices.

For single stock's price:

ds= μs dt + σsdz ---------------------------(i)

μ = Instantaneous rate of return on reckless single stock's price.

S= Price of single stock

σ = volatility or S.D of single stock's price.

dz = Infinitesimal change in price of single stock at instant time t.

z = denotes weiner- Brownian motion in movement of stocks price.

dt 0, dz = $\sqrt{dt}$, $(dz)^2$ = dt ≅ , dzdt ≅ 0

$(dt)^2$ ≅ 0

$(ds)^2 = \mu^2 s^2 (dt)^2 + 2\sigma s^2 dtdz + \sigma^2 s^2$

$(dz)^2 = \sigma^2 s^2 dt$

For two stock's equation

$ds_1 = \mu\, s_1\, dt + 2\sigma_1\, s_1 dz_1$

$ds_2 = \mu\, s_2\, dt + 2\sigma_2\, s_2 dz_2$

Where,

S_1 = Price of underlying stock 1

S_2 = Price of underlying stock 2

T = time remaining before expiration

σ_1 and σ_2 bet he volatility in prices of stocks 1 and 2 respectively.

dz_1 = infinitesimal change in price of stock 1 at instant time t.

dz_2 = Infinitesimal change in the change in price of stock 2 at time instant t.

R = is low risk return rate, and constant until maturity.

z_1 and z_2 denotes the Brownian motion in movement of prices of stock-1 and stock-2 respectively.

$ds_1^2 = \sigma_1^2 s_1^2\, dt$

$ds_2^2 = \sigma_2^2 s_2^2\, dt$

$ds_1\, ds_2 = s_1\, s_2\, \sigma_1\, \sigma_2\, \rho d$

Definition # 2 Delta Hedge Portfolio:

Hedge: It is the strategy to reduce the risk associated with price movements in the underlying price of stock. It is used to remove risk in the portfolio for trading options.

$$\Delta=\frac{\Delta c}{\Delta s}=\frac{\partial c}{\partial s}$$

Δc $= small\ change\ in\ call\ option\ price\ of\ stock$

$\Delta s = small\ change\ in\ price\ of\ stock$

It is used to remove risk in the portfolio for trading options by using strategy to reduce the risk associated with price movements in the underlying price of stock is.

$\Delta=\frac{\Delta c}{\Delta s}=\frac{\partial c}{\partial s}$ (call option pricing)

$\Delta=\frac{\Delta p}{\Delta s}=\frac{\partial p}{\partial s}$ (Put option pricing)

Delta hedges portfolio for single stock defined as.

$\pi = c - \Delta s$

$$\pi = C-\left(\frac{\partial c}{\partial s}\right)s = C - s\frac{\partial c}{\partial s}$$

$$\pi = d\pi + \frac{\partial c}{\partial s}ds$$

$$dc = d\pi + \frac{\partial c}{\partial s}ds$$

$D\pi = \pi r dt$

Delta Hedges portfolio for two stock defined as

π=c- Δs_1- Δs_2 $\therefore \Delta = \left(\frac{\partial c}{\partial s}\right)$ $\pi = c - s_1\left(\frac{\partial c}{\partial s_1}\right) - s_1\left(\frac{\partial c}{\partial s_2}\right)$

$d\pi = dc - \frac{\partial c_1}{\partial s_1}ds_1 - \frac{\partial c_2}{\partial s_1}ds_2$ and

$d\pi = \pi dt$ ($\therefore$ r = risk free ratio of interest)

Definition Ito's lemma for price of single stock

Consider function

f = f(x)

Δf = error in approximation of f

$$\Delta f = \frac{\partial f}{\partial x}\Delta x \quad ; \Delta x \;\; error\ in\ approximation\ of\ f$$

$$df = \frac{\partial f}{\partial x}dx$$

To get more accuracy

$$df = \frac{1}{1!}\frac{\partial f}{\partial x}dx + \frac{1}{2!}\frac{\partial^2 f}{\partial x^2}(dx)^2 + \frac{1}{3!}\frac{\partial^2 f}{\partial x^3}(dx)^2 + \dots$$

Consider c= c(s, t) = call option price of single stock

$$dc = \frac{\partial c}{\partial s}ds + \frac{\partial c}{\partial t}dt + \frac{1}{2}\frac{\partial^2 f}{\partial s^2}(ds)^2$$

$$dc = \frac{\partial c}{\partial s}ds + \frac{\partial c}{\partial t}dt + \frac{1}{2}\sigma^2 s^2\frac{\partial^2 f}{\partial s^2}$$

Be the Ito's Lemma for single stock.

Definition: Ito's Lemma for price of two stocks

Consider c= c (s_1, s_2, t) = call option price of two stocks

$$\left\{\frac{\partial c}{\partial s_1}ds_1 + \frac{\partial c}{\partial s_2}ds_2 + \frac{\partial c}{\partial t}dt + \frac{1}{2}\frac{\partial^2 c}{\partial s_1^2}ds_1^2 + \frac{1}{2}\frac{\partial^2 c}{\partial s^2}ds_2^2 + \frac{\partial^2 c}{\partial s_1 \partial s_2}ds_1 ds_1\right\}$$

By Geometric Brownian motion.

ds_1^2 - $\sigma_1^2 s_1^2$ dt

ds_2^2 - $\sigma_2^2 s_2^2$ dt

$ds_1\, ds_2$= $s_1\, s_2\, \sigma_1\, \sigma_2\, \rho dt$

$$dc = \frac{\partial c}{\partial s_1}ds_1 + \frac{\partial c}{\partial s_2}ds_2 + \frac{1}{2}\sigma_1^2 s_1^2\frac{\partial^2 c}{\partial s_1}dt + \frac{1}{2}\sigma_2^2 s_2^2\frac{\partial^2 c}{\partial s_2}dt + \rho\sigma_1\sigma_2 s_1 s_2\frac{\partial^2 c}{\partial s_1 \partial s_2}dt$$

Derivation of Time fraction order black Sholes model for single stock

Consider a function

$$\Delta f = f = f(x) \text{ ---------------------- } (i)$$

Δf=Error in approximation of f

$$\Delta f = \frac{df}{dx}\Delta x \text{ ------------- } (ii)$$

$$\Delta f = \frac{df}{dx}dx \text{ ------------- } (iii)$$

To get more accuracy,

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial^2 f}{\partial x^2}\frac{dx}{2!} + \frac{\partial^3 f}{\partial x^3}\frac{dx}{3!} + \dots$$

Consider c=(s, t) = Call option price of stock

S= Price of share of stock

t= units of time

$$dc = \frac{\partial c}{\partial s}ds + \frac{\partial c}{\partial t}dt + \frac{1}{2}\frac{\partial^2 f}{\partial s^2}(ds)^2 \text{ ------ } (iv)$$

To obtain single model for single stock, we need three major ingredients

(i) Geometric motion in Brownian movement.

(ii) Ito's Lemma.

(iii) Delta hedging portfolio.

The geometric Brownian motion for share price of stock is

$$ds = \mu s dt + \sigma s dz$$

Where z denotes Weiner Brownian motion

Here, $\mu = instantaneous\ rate\ of\ return\ of\ rickless\ share\ price\ of\ stock.$ $\sigma = volatility\ of\ stock\ price\ at\ instant\ time\ t$

$$dz = \sqrt{dt} \qquad dt \to 0$$

dz^2=dt;dz^2=0

$$dzdt = 0$$

$z\ shows\ infintesimal\ change\ in\ the\ time\dots\dots$

$$ds^2 = \mu^2 s^2 (dt)^2 + 2\mu s\sigma dt dz + \mu^2 s^2 dz^2$$

$$ds^2 = \mu^2 s^2 dt$$

Delta hedges Portfolio defined as:

$$\pi = c - \Delta s \qquad \therefore \Delta = \frac{\partial c}{\partial s}$$

$$\pi = c - s\frac{\partial c}{\partial s}$$

$$d\pi = dc - \frac{\partial c}{\partial s}ds$$

$$dc = d\pi + \frac{s\partial c}{\partial s}ds$$

Eqn. (4) representing Ito's lemma

$$dc = s\frac{\partial c}{\partial s}ds = \frac{\partial c}{\partial s}ds + \frac{\partial c}{\partial t}dt + \frac{1}{2}\sigma^2 s^2\frac{\partial^2 c}{\partial s^2}dt$$

$$d\pi = \frac{2c}{\partial c}dt + \frac{1}{2}\sigma^2 s^2\frac{\partial^2 c}{\partial s^2}dt$$

$$\pi r dt = \frac{2c}{\partial c}dt + \frac{1}{2}\sigma^2 s^2\frac{\partial^2 c}{\partial s^2}dt$$

$$\pi r = \frac{2c}{\partial c} + \frac{1}{2}\sigma^2 s^2\frac{\partial^2 c}{\partial s^2}$$

$$(c - \Delta s)r = \frac{\partial c}{\partial t} + \frac{1}{2}\sigma^2 s^2\frac{\partial^2 c}{\partial s^2}$$

$$\left(c - s\frac{\partial c}{\partial t}\right) r = \frac{\partial c}{\partial t} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 c}{\partial s^2}$$
$$\frac{\partial c}{\partial t} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 c}{\partial s^2} + rs\frac{\partial c}{\partial t} - rc = 0$$

The time dependent fractional order for BS-Model for single stock of call option pricing called as.

$$\frac{\partial^{\propto} c}{\partial t^{\propto}} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 c}{\partial s^2} + rs\frac{\partial c}{\partial t} - rc = 0$$

Subject to maximum pay-off: c (s, t) = max (f(s)-k), 0)

$$\frac{\partial^{\propto} c}{\partial t^{\propto}} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 c}{\partial s^2} + rs\frac{\partial c}{\partial t} - rc = 0$$

Derivation of Time Fractional order Black Sholes Model for two stocks.

Consider a function

$$ds = \mu s dt + \sigma s dz$$

Where,

$\mu = Instanteneous\ rate\ of\ return\ on\ riskless\ as$

$\sigma = volatility\ of\ stock$

$dz = Infinitesimal\ rate\ change\ in\ price\ of\ stock$

$$c = f(x)$$

Consider Δf = Error in approximation of f

$\Delta x =$ Error in x

$$\Delta c = \frac{dc}{dx}\Delta x$$

To get more accuracy

$$\Delta c = \frac{df}{dx}\Delta x + \frac{d^2 f}{dx^2}\frac{(\Delta x)^3}{2!} + \frac{d^3 f}{dx^3}\frac{(\Delta x)^3}{3!} + \ldots$$

If c=f (x, t)

$$dc = \left\{\frac{\partial c}{\partial s_1}ds_1 + \frac{\partial c}{\partial s_2}ds_2 + \frac{\partial c}{\partial t}dt + \frac{1}{2}\frac{\partial^2 c}{\partial s_1^2}(ds_1^2) + + \frac{1}{2}\frac{\partial^2 c}{\partial s^2}(ds^2) + \frac{\partial^2 c}{\partial s_1 \partial s_2}ds_1 ds_2\right\} -- (i)$$

To obtain Black Scholes Model for two stocks, we need the following major ingredients.

(i) Geometric Brownian motion movement.
(ii) Ito's Lemma.
(iii) Delta Hedge portfolio.

Geometric Brownian's Motion for share price of stock is

$$ds_1 = \mu s_1 dt + \sigma_1 s_1 dz_1$$
$$ds_2 = \mu s_2 dt + \sigma_2 s_2 dz_2$$

Where z_1 and z_2 denotes Weiner-Brownian Motion:

Delta hedge Portfolio:

$$\pi = c - \Delta s_1 - \Delta s_2$$
$$\pi = c - \left(\frac{\partial c}{\partial s_1}\right)s_1 - \left(\frac{\partial c}{\partial s_2}\right)s_2 \quad --- (i)$$
$$\pi = c - s_1\frac{\partial c}{\partial s_1} - s_2\frac{\partial c}{\partial s_2} \quad --- (ii)$$
$$d\pi = dc - \frac{\partial c}{\partial s_1}\Delta s_1 - \frac{\partial c}{\partial s_2}\Delta s_2 \quad --- (iii)$$
$$d\pi = \pi r dt \quad --- (iv)$$
$$dt \to 0$$
$$dz = \sqrt{dt} \ and\ (dz)^{\wedge}2 = dt$$
$$dzdt \cong 0 \ and\ (dz)^2 \cong 0$$
$$ds_1^2 = \sigma_1^2 s_1^2 dt^2 + \mu s_1^2 \sigma_1 dz_1 dt + \sigma_1^2 s_1^2 dz_1^2 = \sigma_1^2 s_1^2 dz_1^2$$
$$ds_2^2 = \sigma_2^2 s_2^2 dt^2 + \mu s_2 \sigma_1 dz_2 dt + \sigma_2^2 s_2^2 dz_2^2 = \sigma_2^2 s_2^2 dz_2^2$$
$$ds_1^2 = \sigma_1^2 s_1^2 dt \quad --- (iv)$$
$$ds_2^2 = \sigma_2^2 s_2^2 dt \quad --- (v)$$
$$ds_1 ds_2 = \rho dt \quad --- (iv)$$

Where ρ = correlation coefficient b/w dz_1 and dz_2

from (iv), (v) and (iv), the Ito's lemma for stocks given as.

$$dc = \left\{\frac{\partial c}{\partial s_1}ds_1 + \frac{\partial c}{\partial s_2}ds_2 + \frac{\partial c}{\partial t}dt + \frac{1}{2}\sigma_1^2 s_1^2 \frac{\partial^2 c}{\partial s_1^2}(dt) + \frac{1}{2}\sigma_2^2 s_2^2 \frac{\partial^2 c}{\partial s_2}dt + \rho\sigma_1\sigma_2 s_1 s_2 \frac{\partial^2 c}{\partial s_1 \partial s_2}\right\} -- (v)$$

By equation (iii)

$$d\pi = dc - \frac{\partial c}{\partial s_1}\Delta s_1 - \frac{\partial c}{\partial s_2}\Delta s_2$$

$$\frac{\partial c}{\partial s_1}ds_1 + \frac{\partial c}{\partial s_2}ds_2 + \frac{\partial c}{\partial t}dt + \frac{1}{2}\sigma_1^2 s_1^2 \frac{\partial^2 c}{\partial s_1^2}(dt) + \frac{1}{2}\sigma_2^2 s_2^2 \frac{\partial^2 c}{\partial s_2}dt + \rho\sigma_1\sigma_2 s_1 s_2 \frac{\partial^2 c}{\partial s_1 \partial s_2}\ dt$$

$$r\pi dt = \frac{\partial c}{\partial t}dt + \frac{1}{2}r_1^2 s_1^2 \varepsilon_1^2 \frac{\partial^2 c}{\partial s_1^2}dt + \frac{1}{2}r_2^2 s_2^2 \varepsilon_2^2 \frac{\partial^2 c}{\partial s_2^2}dt + \varepsilon_1\varepsilon_2\sigma_1\sigma_2 \frac{\partial^2 c}{\partial s_1 \partial s_2}dt$$

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$$r\left(c - s_1\frac{\partial c}{\partial s_1} - s_2\frac{\partial c}{\partial s_2}\right) = \frac{1}{2}\sigma_1^2 s_1^2\frac{\partial^2 c}{\partial s_1^2} + \frac{1}{2}\sigma_2^2 s_2^2\frac{\partial^2 c}{\partial s_2^2} + \rho\sigma_1\sigma_2 s_1 s_2\frac{\partial^2 c}{\partial s_1\partial s_2} + \frac{\partial c}{\partial t}$$

$$\frac{\partial c}{\partial t} + \frac{1}{2}\sigma_1^2 s_1^2\frac{\partial^2 c}{\partial s_1^2} + \frac{1}{2}\sigma_2^2 s_2^2\frac{\partial^2 c}{\partial s_2^2} + rs_1\frac{\partial c}{\partial s_1} + rs_2\frac{\partial c}{\partial s_2} + \rho\sigma_1\sigma_2 s_1 s_2\frac{\partial^2 c}{\partial s_1\partial s_2} - rc = 0 --- (i)$$

Be the BS model for European style call option pricing for two stocks.

The fractional order time dependent black sholes P.D.E model for call option pricing of two stocks is represented as below.

$$\frac{\partial^r c}{\partial t^r} + \frac{1}{2}\sigma_1^2 s_1^2\frac{\partial^2 c}{\partial s_1^2} + \frac{1}{2}\sigma_2^2 s_2^2\frac{\partial^2 c}{\partial s_2^2} + + rs_1\frac{\partial c}{\partial s_1} + rs_2\frac{\partial c}{\partial s_2} + \rho\sigma_1\sigma_2 s_1 s_2\frac{\partial^2 c}{\partial s_1\partial s_2} - rc = 0$$

Subject to maximum pay off.

$$c(x,y,t) = (w_1 f(s_1) + w_2 f(s_2) - k, 0)$$

Similarly,

The fractional order time dependent Black-Sholes P.D.E Model for call option pricing Model is represented as.

$$\frac{\partial^r p}{\partial t^r} + \frac{1}{2}\sigma_1^2 s_1^2\frac{\partial^2 p}{\partial s_1^2} + \frac{1}{2}\sigma_2^2 s_2^2\frac{\partial^2 p}{\partial s_2^2} + + rs_1\frac{\partial p}{\partial s_1} + rs_2\frac{\partial p}{\partial s_2} + \rho\sigma_1\sigma_2 s_1 s_2\frac{\partial^2 p}{\partial u\partial v} - rp = 0$$

Subject to maximum pay-off.

$$p(x,y,t) = max\,(K - (w_1 f(s_1) + w_2 f(s_2), 0)$$

$$\frac{\partial^r p}{\partial t^r} + \frac{1}{2}\sigma_1^2 s_1^2\frac{\partial^2 p}{\partial s_1^2} + \frac{1}{2}\sigma_2^2 s_2^2\frac{\partial^2 p}{\partial s_2^2} + + rs_1\frac{\partial p}{\partial s_1} + rs_2\frac{\partial p}{\partial s_2} + \rho\sigma_1\sigma_2 s_1 s_2\frac{\partial^2 p}{\partial u\partial v} - rp = 0$$

The Black and Scholes is formed financial mathematical differential equation of first order and one dimension involves

C = call option price

S = stock price

R = Risk free rate

$\sigma = standard\ derivation$ Or volatility of stock returned.

T = time before expiring date.

$$\frac{\partial^{\propto} c}{\partial t^{\propto}} + \frac{1}{2}\sigma^2 s^2\frac{\partial^2 c}{\partial^2 s} + rs\frac{\partial c}{\partial s} - rc = 0$$

Finite volume method For Unsteady flow

$$\int_t^{t+\Delta t} \frac{\hat{}\ /}{t\hat{}\propto} dvdt + \frac{1}{2}\sigma^2 \int_t^{t+\Delta t} \frac{\hat{}\ /}{s\hat{}2} dvdt + r\int_t^{t+\Delta t} \frac{c/}{t} vdt - \_t\hat{}t + \Delta trcdvdt = 0$$

$$\int_t^{t+\Delta t} \frac{\hat{}\ /}{t\hat{}\propto} dvdt + \frac{1}{2}\sigma^2 \int_t^{t+\Delta t} \frac{/}{S}\left(\frac{c/}{s}\right) vdt + r\_t\hat{}t + \Delta t\frac{\partial c}{\partial t} dvdt - \int_t^{t+\Delta t} cdvdt = 0$$

C_P-?C_P^0/Γ2-∝ΔV+1/2σ^2 _T^T+ΔT∂C/∂SA|_E-∂C/∂SA|_Wdt +r AC ?|_t^t+Δt-rCA?|_t^t+Δt=0=0;

$$\frac{C_P - C_P^0}{(\Gamma 2 - \propto)}\Delta V + \frac{1}{2}\sigma^2\left[\int_T^{T+\Delta T} \left(\frac{C/}{S}\right) - \frac{\partial C}{\partial S}A|_W\right]dt = 0 = 0\ ;------ (2)$$

Flux across through all faces

Flux through west face = $A_W\frac{\partial C}{\partial S}|_W$ = $A_W\left(\frac{C_p - C_W^0}{\delta_X W_P}\right)$ .... (A)

Flux through east face = $A_E\frac{\partial C}{\partial S}|_E$ = $A_E\left(\frac{C_E^0 - C_p}{\delta_X P_E}\right)$ .... (B)

NOW SUBSTITUTING A AND B INTO (2)

$$\frac{C_P - C_P^0}{(\Gamma 2 - \propto)}\Delta V + \frac{1}{2}\sigma^2\left[\left(\frac{C_E^0 - C_p}{\delta_X P_E}\right) - \left(\frac{C_p - C_W^0}{\delta_X W_P}\right)\right]\Delta t + = 0;........$$

$$\left[\frac{A}{(\Gamma 2 - \propto)}\Delta X - \frac{1}{2}\sigma^2\frac{\Delta T}{\delta_X P_E} - \frac{1}{2}\sigma^2\frac{\Delta T}{\delta_X W_P}\right] C_P + \frac{1}{2}\sigma^2\frac{\Delta T}{\delta_X P_E} C_E^0 + \frac{1}{2}\sigma^2\frac{\Delta T}{\delta_X W_P} C_W^0 = 0............. (3)$$

Here $a_p = \left[\frac{A}{(\Gamma 2 - \propto)}\Delta X - \frac{1}{2}\sigma^2\frac{\Delta T}{\delta_X P_E} - \frac{1}{2}\sigma^2\frac{\Delta T}{\delta_X W_P}\right]$

$$a_e = \frac{1}{2}\sigma^2\frac{\Delta T}{\delta_X P_E}$$

$$a_w = \frac{1}{2}\sigma^2\frac{\Delta T}{\delta_X W_P}$$

NOW THE EQUATION (3) BECOMES

$$a_p\ C_p + a_e C_E^0 + a_w c_w^0 = 0$$

THE EXPLICIT FORM IS.

$$a_p\ C_p = -a_e C_E^0 - a_w c_w^0$$

$$C_p = -\frac{a_e}{a_p} C_E^0 - \frac{a_w}{a_p} c_w^0$$

Applying Finite Volume method to evaluate European call option price from two stoke Fractional B-S Model:

Consider below Fractional order European call option pricing P.D.E for two stocks.

$$\frac{\partial^\propto c}{\partial t^\propto}+\frac{\sigma_1^2}{2}s_1^2\frac{\partial^2 c}{\partial s_1^2}+\frac{\sigma_2^2}{2}s_2^2\frac{\partial^2 c}{\partial s_2^2}+rs_1\frac{\partial c}{\partial s_1}+rs_2\frac{\partial c}{\partial s_2}+\rho\, s_1 s_1\sigma_1\sigma_2\frac{\partial^2 c}{\partial s_1\partial s_2}-rC=0\text{......... (1)}$$

Subject to pay-off to the investor,

$$C(s_1,s_2,t)=(w_1\ s_1+ws_2-k,0)$$

Where

s_1 = Price of the share of stock1

s_2 = Price of the share of stock2

ρ = Correlation coefficient between stock1 and stock2

σ_1= Volatility price or S.D of stock1

σ_2= Volatility price or S.D of stock2

K = Strike price or Exercise price for call option

w_1= Properties of investment on stock1

w_2= Properties of investment on stock2

Equation (2) can be simplified by considering the substitution.

$$\frac{\partial^\alpha C}{\partial t^\alpha}+\frac{\sigma_1^2}{2}\frac{\partial^2 C}{\partial s_1{}^2}+\frac{\sigma_2^2}{2}\frac{\partial^2 C}{\partial s_2^2}+\rho\sigma_1\sigma_2\frac{\partial C}{\partial s_1\partial s_2}-rC=0 \quad (2)$$

BY APPLYING THE FINITE VOLUME METHOD FOR UNSTEADY FLOW

$$\frac{\partial^\alpha C}{\partial t^\alpha}+\frac{\sigma_1^2}{2}\frac{\partial^2 C}{\partial s_1{}^2}+\frac{\sigma_2^2}{2}\frac{\partial^2 C}{\partial s_2^2}+\rho\sigma_1\sigma_2\frac{\partial C}{\partial s_1\partial s_2}-rC=0$$

$$\int_{CV}\int_{\wedge}^{+\Delta T}\frac{\alpha^{C}\partial}{T^\wedge\alpha}DVDT+\frac{1}{2}\sigma_1^2\int_{CV}\int_{\wedge}^{+\Delta T}\frac{2^{C}\partial}{?S\_1^\wedge 2}DVDT+\frac{1}{2}\sigma_2^2\int_{CV}\int_{\wedge}^{+\Delta T}\frac{2^{C}\partial}{?S\_2^\wedge 2}DVDT+P\sigma_1\sigma_2\int_{CV}\int_{\wedge}^{+\Delta T}\frac{2^{C}\partial}{S\_1\partial S_2}DVDT-\int_{CV}\int_{\wedge}^{+\Delta T}DVDT=0$$

_CV^._T^T+ΔT∂^AC/∂T^ADVDT+1/2?Σ_1^2_CV^._T^T+ΔT∂/∂S_1(∂C/∂S_1)DVDT+

$$\frac{1}{2}\sigma_2^2\int_{CV}\int_{\wedge}^{+\Delta T}\frac{\partial}{S\_2}\left(\frac{\partial C}{\partial S_2}\right)DVDT+P\sigma_1\sigma_2\int_{CV}\int_{\wedge}^{+\Delta T}\frac{\partial}{S\_1}\left(\frac{\partial C}{\partial S_2}\right)DVDT-\int_{CV}\int_{\wedge}^{+\Delta T}DVDT=0;\ \frac{C_P-C_{P0}}{(\Gamma 2-\propto)}\Delta V+$$

$$\frac{1}{2}\sigma_1^2\left[\int_T^{T+\Delta T}\left(\frac{C/}{S\_1}A\right)|_E-\left(\frac{\partial C}{\partial S_1}A\right)|_W\right]dt+\frac{1}{2}\sigma_2^2\left[\int_T^{T+\Delta T}\left(\frac{C/}{S\_2}A\right)|_N-\left(\frac{\partial C}{\partial S_2}A\right)|_S\right]dt+\rho\sigma_1\sigma_2\left[\int_T^{T+\Delta T}\left(\frac{C/}{S\_2}A\right)|_N-\left(\frac{\partial C}{\partial S_2}A\right)|_S\right]dt-\int_t^{t+\Delta t}c\Delta v dt=0$$

C_P-C_P^0/Γ2-∝ΔV+1/2?Σ_1^2 _T^T+ΔT∂C/∂S_1|_EA_E-∂C/∂S_1|_WA_Wdt

$$+\frac{1}{2}\sigma_2^2\left[\int_T^{T+\Delta T}\frac{C/}{S\_2}|_N A_N-\frac{\partial C}{\partial S_2}|_S A_S\right]dt+\rho\sigma_1\sigma_2\left[\int_T^{T+\Delta T}\frac{C/}{S\_2}|_N A_N-\frac{\partial C}{\partial S_2}|_S A_S\right]dt-\int_t^{t+\Delta t}c\Delta v dt=0(3)$$

Flux across through all faces

Flux through west face = $A_W\frac{\partial C}{\partial S_1}|_W$ = $A_W\left(\frac{C_P-C_W^0}{\delta_X W_P}\right)$ (A)

Flux through south face = $A_S\frac{\partial C}{\partial S_2}|_S$ = $A_S\left(\frac{C_P-C_S^0}{\delta_Y S_P}\right)$ (B)

Flux through north face = $A_N\frac{\partial C}{\partial S_2}|_N$ = $A_S\left(\frac{C_N^0-C_P}{\delta_Y P_N}\right)$ (C)

Flux through east face = $A_E\frac{\partial C}{\partial S_1}|_E$ = $A_E\left(\frac{C_E^0-C_P}{\delta_X P_E}\right)$ (D)

NOW SUBSTITUTING (A), (B), (C) AND (D) INTO (2)

$$\frac{C_P-C_{P0}}{(\Gamma 2-\propto)}\Delta V+\frac{1}{2}\sigma_1^2\left[\left(\frac{C_E^0-C_P}{\delta_X P_E}\right)-\left(\frac{C_P-C_W^0}{\delta_X W_P}\right)\right]\Delta t+\frac{1}{2}\sigma_2^2\left[\left(\frac{C_N^0-C_P}{\delta_Y P_N}\right)-\left(\frac{C_P-C_S^0}{\delta_Y S_P}\right)\right]\Delta t+\rho\sigma_1\sigma_2\left[\left(\frac{C_N^0-C_P}{\delta_Y P_N}\right)-\left(\frac{C_P-C_S^0}{\delta_Y S_P}\right)\right]\Delta t-rc\Delta v\Delta t=0$$

$$\frac{C_P-C_{P0}}{(\Gamma 2-\propto)}\Delta V+\frac{1}{2}\sigma_1^2\left[\left(\frac{C_E^0-C_P}{\delta_X P_E}\right)-\left(\frac{C_P-C_W^0}{\delta_X W_P}\right)\right]\Delta t+\left(\frac{1}{2}\sigma_2^2+\rho\sigma_1\sigma_2\right)\left[\left(\frac{C_N^0-C_P}{\delta_Y P_N}\right)-\left(\frac{C_P-C_S^0}{\delta_Y S_P}\right)\right]\Delta t-rc\Delta v\Delta t=0$$

$$[(\frac{A}{(\Gamma 2-\propto)}\Delta X-\frac{1}{2}\sigma_1^2\frac{\Delta T}{\delta_X P_E}-\frac{1}{2}\sigma_1^2\frac{\Delta T}{\delta_X W_P}-\left(\frac{1}{2}\sigma_2^2+\rho\sigma_1\sigma_2\right)\frac{\Delta T}{\delta_Y P_N}-\left(\frac{1}{2}\sigma_2^2+\rho\sigma_1\sigma_2\right)\frac{\Delta T}{\delta_Y S_P}]C_P+\frac{1}{2}\sigma_1^2\frac{\Delta T}{\delta_X P_E}C_E^0+\frac{1}{2}\sigma_1^2\frac{\Delta T}{\delta_X W_P}C_W^0+\left(\frac{1}{2}\sigma_2^2+\rho\sigma_1\sigma_2\right)\frac{\Delta T}{\delta_Y P_N}C_N^0+\left(\frac{1}{2}\sigma_2^2+\rho\sigma_1\sigma_2\right)\frac{\Delta T}{\delta_Y S_P}C_S^0-rc\Delta v\Delta t=0 \quad (3)$$

Here we suppose

$$a_p=[\frac{A}{(\Gamma 2-\propto)}\Delta X-\frac{1}{2}\sigma_1^2\frac{\Delta T}{\delta_X P_E}-\frac{1}{2}\sigma_1^2\frac{\Delta T}{\delta_X W_P}-\left(\frac{1}{2}\sigma_2^2+\rho\sigma_1\sigma_2\right)\frac{\Delta T}{\delta_Y P_N}-\left(\frac{1}{2}\sigma_2^2+\rho\sigma_1\sigma_2\right)\frac{\Delta T}{\delta_Y S_P}]$$

$$a_e=\frac{1}{2}\sigma_1^2\frac{\Delta T}{\delta_X P_E} \qquad a_w=\frac{1}{2}\sigma_1^2\frac{\Delta T}{\delta_X W_P}$$

$$a_s = \left(\frac{1}{2}\sigma_2^2 + \rho\sigma_1\sigma_2\right)\frac{\Delta T}{\delta_Y S_P} \qquad a_n = \left(\frac{1}{2}\sigma_2^2 + \rho\sigma_1\sigma_2\right)\frac{\Delta T}{\delta_Y P_N}$$

AND S = $rc\Delta v\Delta t$.

NOW THE EQUATION (3) BECOMES

$$a_p\ C_p + a_e C_E^0 + a_w c_w^0 + a_n c_n^0 + a_s c_s^0 - s = 0$$

THE EXPLICIT FORM IS.

$$a_p\ C_p = s - a_e C_E^0 - a_w c_w^0 - a_n c_n^0 - a_s c_s^0$$

$$C_p = \frac{s}{a_p} - \frac{a_e}{a_p} C_E^0 - \frac{a_w}{a_p} c_w^0 - \frac{a_n}{a_p} c_n^0 - \frac{a_s}{a_p} c_s^0$$

- **RESULT**
- **DISCUSSION & RECOMMENDATION**

Exercise price of stock1 = 1
Exercise price of stock2 = 2
Maximum Exercise price = k
Month of Expiration time = t
Strategy: Call option.
1 = S.D of stock1
2 = S.D of stock2
w1= Proportion of stock1
w2= Proportion of stock1
r = Risk free rate of return
= Correlation coefficient between Stock1 and stock2
Given:
σ_1 = 0.40, σ_2= 0.20,
r =8%, = 0.75, = 60,
w_1 = 1, w_2 = 3; ∝ = 1, 0.005, 0.25, 0.5 and 0.75

Time= t =5 months I.C:
(, , 0) = R .65
Boundary Conditions (x, y are in Rs.)
C = Rs.70 at x=0; 0≤ ≤80
C = Rs.60 at x=60; 0≤ ≤80
C = Rs.50 at y=0; 0≤ ≤60
C = Rs.20 at x=80; 0≤ ≤60

Using the Finite Volume Iterative Tri-Diagonal Algorithm Method (TDMA) to estimate call option pricing:

- Setting Network of the grid and locate Node or Mesh points.
- For our convenience We scaling 0≤ ≤0.6, 0≤ ≤0.8
- $h_1 = \frac{\sigma_1^2}{2}$, $h_2 = \frac{\sigma_2^2}{2}$, $h_3 = -\rho\sigma_1\sigma_2$
- C = 0.7 at x=0 ; 0 ≤ ≤ 0.8
- C = 0.6 at x=0.6 ; 0 ≤ ≤ 0.8
- C = 0.5 at y=0 ; 0 ≤ ≤ 0.6
- C = 0.2 at x=0.8 ; 0 ≤ ≤ 0.6
- Applying finite volume method At node 1 to node 12
- We get the result mentioned in table

Nodes	a_p	a_n	a_s	a_e	a_w	
1	0.3120	0.08	0	0.08	0	0.2117
2	0.3120	0.08	0.08	0.08	0	0.2032
3	0.3120	- 0.08	0.08	0.08	0	0.2032
4	0.3120	0.08	0.08	0.08	0	0.1998
5	0.3120	0.08	0	0.08	0.08	0.2005
6	0.1472	0.08	0.08	0.08	0.08	0.1920
7	0.1472	0.08	0.08	0.08	0.08	0.1920
8	0.3120	0.08	0.08	0.08	0.08	0.1886
9	0.3120	0.08	0	0	0.08	0.1845
10	0.3120	0.08	0.08	0	0.08	0.1760
11	0.3120	0.08	0.08	0	0.08	0.1760
12	0.3120	0.08	0.08	0	0.08	0.1726

Nodes	Cj (Rs.)
1	91.58
2	83.24
3	10.932
4	30.6391
5	82.92
6	71.85
7	72.16
8	92.26
9	68.96
10	68.54
11	68.50
12	64.39

At $\propto$ = 1

Price Of Stock 2 in (Rs.)	Price Of Stock 1 in (Rs.)		
	10	30	50
10	0.9158	0.8292	0.6896
30	0.8324	0.7185	0.6854
50	1.0932	0.7216	0.6850
70	30.6391	0.9226	0.6439

The table illustrates the greatest profit that may be made by selling stock1 at 10 and stock2 at 70. If the asset one is sold for 50 and the asset two is sold at 70, the values for loss can also be shown in the following table.

At $\propto$ = 0.005

Price Of Stock 2 in (Rs.)	Price Of Stock 1 in (Rs.)		
	10	30	50
10	1.3571	1.1239	0.7600
30	1.1445	0.8490	0.7470
50	1.4562	0.8535	0.7444
70	15.1474	1.0047	0.6365

The table shows that selling the stock1 at 10 and selling the stock2 at 70 yields the highest profit. If the asset one is sold for 50 and the asset two is resold at 70, the values for loss can also be shown in the following table.

DISCUSSION

"Bai Al Arboun" can actually become a basis for developing Shariah compliant alternative to conventional call Options.

In this study, I substituted two assets for the time-fractional Black-Scholes PDE in the Islamic society. Finite-Volume approach (unsteady flow) gives successful and satisfactory numerical solutions to 2-dimensional B-S time-fractional ordered Partial Differential Equation for two stocks subjected to any I.B.V issue by constructing controlled finite volume grids at each nodal point. For unsteady flow, the solutions of Black-Scholes PDE is efficiently and successfully calculated using the simpler Finite-Volume approach in the least amount of time and with the least amount of computation. A practical example is also provided to demonstrate the suggested scheme's dependability, efficiency, simplicity, and efficacy.

RECOMMENDATION

These assumptions reflect a perfect market are far from being realistic, hence the need to seek a more appropriate model for the pricing of the deposit amount "Al Arboun" that takes into consideration the assumptions of compliance with Shariah rules and the fact that "Al Arboun" is closer to an American option rather than an European option since the buyer could exercise the option at any time prior to maturity.

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