Spectrum of Engineering Sciences
ISSN (e) 3007-3138 (p) 3007-312X Volume 4, Issue 4, 2026

CUBIC HESITANT FUZZY SOFT SET AND ITS APPLICATION TO MULTI-CRITERIA DECISION-MAKING

*1Faizan Ahmad, [2]Dr. Muhammad Fiaz, [3]Maqsood Ur Rahman, [4]Shoaib Qaiser, [5]Sunihri Hameed, [6]Abdur Rehman, [7]Adil Nadeem

*1BS Graduated, Department of Mathematics Government Postgraduate College Karak, 27200, KP Pakistan.

[2]Lecturer in Mathematics, Department of Mathematics, Government Post Graduate College Karak 27200, KP, Pakistan.

[3]Associate Professor, Department of Mathematics, Government Post Graduate College Karak 27200, KP, Pakistan.

[4]BS Gratuated, Department of Mathematics, Government Post Graduate College Karak 27200, KP, Pakistan.

[5]BS Student, Department of Mathematics, Government Post Graduate College Karak 27200, KP, Pakistan.

[6]BS Student, Department of Mathematics, Government Post Graduate College Karak 27200, KP, Pakistan.

[7]BS Student, Department of Mathematics, Government Post Graduate College Karak 27200, KP, Pakistan.

*1faizanft00@gmail.com, [2]fiazktk03@gmail.com, [3]mkhattak7070@gmail.com, [4]shoaibqaiserktk1998@gmail.com, [5]Sunihrihameed@gmail.com, [6]abdurehman0987543@gmail.com, [7]adilnadeem02@gmail.com

Keywords
Cubic Hesitant Fuzzy Soft Set; Aggregation Operators; Multi-Criteria Decision-Making; Cubic Fuzzy Set; Hesitant Fuzzy Soft Set

Article History
Received on 29 March, 2026
Accepted on 15 April, 2026
Published on 16 April, 2026

Corresponding Author:
Faizan Ahmad

Abstract

In this paper, a novel concept of Cubic Hesitant Fuzzy Soft Sets (Cubic HFSSs) is introduced by integrating the theories of cubic sets and hesitant fuzzy soft sets. Fundamental operational laws of Cubic HFSSs are developed, and their essential properties are thoroughly investigated. Furthermore, several new aggregation operators are proposed, including the Cubic Hesitant Fuzzy Soft Weighted Averaging (Cubic HFSWA), Cubic Hesitant Fuzzy Soft Weighted Geometric (Cubic HFSWG), Cubic Hesitant Fuzzy Soft Ordered Weighted Averaging (Cubic HFOWA), Cubic Hesitant Fuzzy Soft Ordered Weighted Geometric (Cubic HFOWG), and Cubic Hesitant Fuzzy Hybrid Averaging and Geometric operators. The mathematical characteristics of these operators are systematically analyzed. The proposed framework provides a unified and simplified structure encompassing fuzzy sets, intuitionistic fuzzy sets, hesitant fuzzy sets, and cubic fuzzy sets. In addition, a multi-criteria decision-making (MCDM) approach based on the developed aggregation operators is formulated within the cubic hesitant fuzzy soft environment. A practical numerical example related to the selection of the most suitable crop is presented to demonstrate the applicability and effectiveness of the proposed method. Sensitivity analysis is conducted to examine the stability of the results. Finally, a comparative analysis is performed to validate the superiority and efficiency of the proposed approach over existing decision-making methods.

1.1 Introduction

Decision-making (DM) is development of indicating the top way of accomplishment in several substitutes depending on exact criteria, typically including the assessment and comparison of various aspects to accomplish the most positive results. Zadeh [1] first suggested the notion of Fuzzy Set (FS) symbolized which expands the concept of a standard set by allowing elements, denoted by the symbol x, to have membership degrees μA(x) ∈[0, 1]. Fuzzy sets allocate a single membership degree to each element, which might be incomplete for modeling conditions where the degree of membership is not precise or varies within a range. By spreading the notion of FS including interval-valued fuzzy sets (IVFS) , Gorzaczany [2] offers and analyses an estimate inference technique that makes it possible to validate verbal DM procedures with useful computer representations. He develops an example to establish the method. Jun [3] describes and studies various concepts linked to external and internal cubic sets (CS), P-(R-intersection), P-(R-union), and P-(R-order), diving into their properties, preservation criteria, and the potential for intersections showing both internal and external abilities. Cubic fuzzy set (CFS) contains (IVFS) and FS.

A hesitant fuzzy set (HFS) is one of the additions of the traditional fuzzy sets that contribute for the demonstration of hesitation or doubt of an element in the membership degree. Torra [4] announces Hesitant Fuzzy Sets (HFS) as an addition of conventional Fuzzy set, stressing their unparalleled clarification, introducing major processes, and looking at their relatedness with intuitionistic fuzzy sets (IFS). After that he demonstrates how the cover of (HFS) resembles to that of (IFS) and tells that the newly planned operations are still regular when applied to this cover. HFS enlarges the concept of FS by allowing the possible set of values of the membership degree of an element x instead of a single value hA(x) ∈[0, 1]. Xia [5] explores HFS, extends FS theory, devises aggregation operators (AOs), studies their connections, and shows practical DM use. Liao and Xu [6] introduce subtraction and division processes for HFS, improving their effectiveness in uncertainty contexts, connecting them to IFS, and extending them to connected concepts. Liao and Xu [7] introduce new AOs tailored for HFS, encircling HFHAA, HFHAG, quasi HFHAA, and quasi HFHAG, indicating their features, and employing them in applied examples for multi-criteria decision-making (MCDM). Liao and Xu's [8] research emphases on engaging HFS to address ambiguity, introducing a set of diverse hybrid weighted aggregation operators AOs, and demonstrating their use in MCDM through the assessment of applicant portfolios. Wei [9]. Explore DM with hesitant interval-valued fuzzy (HIVFS) data, introducing facilitators and demonstrating their use with an illustrative example. Interval-valued Hesitant FSs spread the notion of Hesitant FS by allowing the membership degree of a component *x* to be a set of possible interval values instead of a single value.

Mehmood [10] introduces cubic hesitant fuzzy sets (CHFS), summarizes their characteristics and operations, and shows their competence in MCDM situations. The CHFS covers an IVHFS and HFS. Mahmood [11] introduces a series of AOs tailored for CHFS, including both averaging and geometric types, and demonstrates their application in determining MCDM problems by selecting the most appropriate substitute following decision makers' (DM) preferences. Molodtsov [12] introduces the fundamental principles of soft set (SS) theory, presenting it as a supple mathematical framework for management uncertainty and unclear objects, emphasizing primary findings, and defining coming research avenues. Maji, P.K. [13] widens the concept of soft expert sets (SES) to encompass fuzzy soft expert sets (FSES), introducing vital operations, applying this concept in DM, and exploring a mapping within (FSES) classes along with its features. Babitha [14] presents a new concept, the HFSS, which chains elements from soft sets and HFSS, outlining essential operations like intersection, union, and complement, and displaying its utility in DM contexts. Muhiuddin [15] presents a collection of cubic soft set (CSS) ideas, encompassing interior and exterior types, R-cubic P-cubic and soft subsets, R-intersection, R-union, P-intersection, P-union and complements, probing into their characteristics, and encompasses their application to BCK/BCI-algebras by introducing CS BCK/BCI-algebras and authenticating their properties. Yang [16] presents the notion of IVFSS, integration IVFSS and SS models, drawing operations like complement, 'AND,' and 'OR,' validating De Morgan's, associative, and distribution laws, and illustrating its application in result problems using numerical examples. Chen [17] presents a new thought, IVHFS preference relations, within the dominion of fuzzy preference structures for management improbability in GDM, expanding HFS to IVHFSS, present aggregation methods for IVHFSS data, and accommodating dissimilarities in DMR views, all revealed through numerical examples. Das [18] introduces HFSS, merging HFS and SS theory, beneficial for management uncertainty, demonstrated with medical diagnosis examples. Akram [19] introduces IVHFNSSs and lengthens TOPSIS for MAGDM, accommodating various evaluations, random weights, and indecision, increasing decision-making. Jyoti Borah's [20, 21] key objective is to introduce and observe operations such as "Union," "Intersection," and four particular operators (O1, O2, O3, O4) in the background of (IVHFSS), examining into their in-built properties. Wei Liang's research emphases on dealing complex DM by proposing a fresh method, the PIVHFS lost and gained supremacy score, which mixes regret theory, interval evidential thought, and an interval projection measure for useful uncertainty management in supplier range for epidemic preclusion products. S. S. Appadoo [22] showed that the AOs proposed by Amin et al. lacks monotonicity, which in turn creates their TOPSIS method unfitting, highlighting an unexplained and puzzling research problem. Marian [23] introduces an estimated inference technique that spreads FS to IVFS, helping proper representation of verbal verdict algorithms and competent computer execution, illustrated through an example.

Zhao [24] established CVD, the principal reason of death in Asia; this review studies epidemiologic characteristics and debates prevention challenges. Zhao et al. [25] highlight CVD as China's primary death basis, arguing epidemiological tendencies and prevention challenges, with undiagnosed problems. Rehman et al. [26] expect CVD trends using NDGM and evaluate expanding times, highlighting imperative prevention needs for policymakers and healthcare providers worldwide.

We oppose that no earlier studies have covered the use of a cubic hesitant fuzzy soft environment, as cited before. To shorten this gap, we will build a cubic hesitant fuzzy soft set (CHFSS) on universal sets U with definite elements set. We have to define operational Laws, accuracy and score functions, as well as accumulation operators and observe their characteristics. Lastly, we will relate our study to solve real-life policymaking difficulties.

1.2. Research Question

Decision-making (DM) plays a vital role in numerous real-world applications. Motivated by existing studies and gaps identified in the literature, this research seeks to address the following questions:

i. How can a rigorous mathematical framework for Cubic Hesitant Fuzzy Soft Sets (CHFSSs) be constructed?

ii. What fundamental operational laws and mathematical properties can be defined and established for CHFSSs?

iii. Which types of aggregation operators can be developed and theoretically validated within the CHFSS framework?

iv. In what way does the proposed CHFSS model generalize and simplify existing concepts such as fuzzy sets, intuitionistic fuzzy sets, hesitant fuzzy sets, and cubic fuzzy sets?

v. How can complex decision-making (DM) problems be effectively solved within the CHFSS environment?

vi. How does the application of CHFSSs to practical scenarios, such as crop selection, compare with existing methods in the literature?

vii. What insights can be gained from sensitivity analysis regarding the robustness, stability, and reliability of the CHFSS-based decision-making approach?

Table 1: List of Symbols/Notations.

X	Fixed set
δξ	M_D
ξ	Fs
ξ=hδξ	FNs
vÎ	N_{MD}
Γ	IVFS
A	Cubic Fuzzy Set (CFS)
S	Score Function
A	Accuracy Function
AOs	Aggregation Operators
CFWAOs	Cubic Fuzzy Weighted Averaging Operators
CFWGOs	Cubic Fuzzy Weighted Geometric operators
ϖ	Hesitant Fuzzy Set
FA	Soft Set
Fṁ	Fuzzy Soft Set

Literature Review

Fuzzy set theory, introduced by Zadeh, has played a significant role in modeling uncertainty and vagueness in real-world problems. Over time, several extensions of fuzzy sets have been developed to better handle imprecise and incomplete information, including intuitionistic fuzzy sets, hesitant fuzzy sets, and cubic fuzzy sets. Intuitionistic fuzzy sets incorporate both membership and non-membership degrees, while hesitant fuzzy sets allow multiple possible membership values to represent hesitation in decision-making. Cubic fuzzy sets further enhance flexibility by combining interval-valued and precise fuzzy information. In parallel, soft set theory has been widely applied to parameterized decision-making problems due to its ability to handle uncertainty without requiring additional constraints. Numerous aggregation operators and decision-making models based on these theories have been proposed and successfully applied in areas such as resource selection, medical diagnosis, and engineering management. However, existing approaches often face limitations when dealing with highly complex and hesitant information simultaneously. These limitations motivate the development of more generalized and robust frameworks, such as cubic hesitant fuzzy soft sets, to improve decision-making accuracy and reliability.Different versions of FS in the Real Range.

2.1. Fuzzy Set:

Definition 1 [1]: A Fuzzy Set ξ on general set X is stated as:

ξ=hδξ| h∈X,δξ∈[0,1] (1)

where δξ:X⟶0,1. The Fuzzy Ns are represented by ξ=hδξ.

Example 1: Let we consider X={1,2,3}.Then; a ξ defined on X is given by ξ=10.2,20.5,30.6.

2.2. FS Operational Laws

ξi=hδξi=i=1,2,...n be FNs:

1. ξ1∨ ξ2=hδξ1∨δξ2;
2. ξ1∧ξ2=hδξ1∧δξ2;
3. ξς1=h1-δξ1;
4. ξ1+ξ2=h1-1-δξ11-δξ2;
5. ξ1×ξ2=hδξ1δξ2;
6. ξ1⊕ξ2=h1,minδξ1+δξ2;
7. ξ1⊖ ξ2=h0,maxδξ1-δξ2;
8. ξ1⊙ξ2=h0,maxδξ1+δξ2-1;
9. ς.ξ1=h1-δξ1ς;
10. ξ1ς=hδξ1ς;

Example 2: Now let us consider X={h1,h2,h3} & ξi=hδξi, ξ1=h10.4, ξ2=h10.5 , ξ3=h10.7=i=1,2,3, be three FNs. The algebraic properties:

1. ξ1∨ ξ2 = hδξ1∨δξ2 =h10.4∨ 0.5=h10.5;
2. ξ1∧ξ2=hδξ1∧δξ2 =h10.4∧0.5 =h10.4;
3. ξς1=h1-δξ1=h11-0.4=h10.6;
4. ξ1+ξ2=h1-1-δξ11-δξ2=h11-1-0.41-0.5;
5. ξ1×ξ2=hδξ1δξ2;
6. ξ1⊕ξ2=h1,minδξ1+δξ2;
7. ξ1⊖ ξ2=h0,maxδξ1-δξ2;
8. ξ1⊙ξ2=h0,maxδξ1+δξ2-1;
9. ς.ξ1=h1-δξ1ς;
10. ξ1ς=hδξ1ς;

2.3. Interval Valued Fuzzy Set (IVFS):

Definition 2 [2]: Consider the collection of all closed subintervals of [0, 1] is T_A= [TAL, TAU] ∈η, where TAL and TAU are expressed as the upper extreme and the lower extreme, respectively. Let X is a nonempty universal set. Then Γ={xψTΓ:x∈X} (2) is termed as IVFS of X, where ψTΓ:X⟶[0,1] be the degree of membership of x in X and ψTΓ=[ψAL,ψAL] be an IVFNs.

2.4. Cubic Fuzzy Set (CFS)

Definition 3[3]: Let X be the nonempty general set. Then the set

A={(x,ψTΓx,ξA(x))/x∈X} (3) is said to be a CFS of X, where ψTΓx be an IVFS in X and ξA(x) is a FS in X. Some basic concept concerning to CFS are defined in the following steps:

i. **Definition:** A CFS A={(x,[ψ-TΓx,ψ+TΓx],ξA(x))/x∈X} is termed as an internal CFS if the term [ψ-TΓx≤ξA(x)≤ψ+TΓx
ii. **Definition:** A CFS A={(x,[ψ-TΓx,ψ+TΓx],ξA(x))/x∈X} is termed as an external CFS if the term ξA(x)∉ (ψ-TΓx,ψ+TΓx) ;

2.5. Operational Laws Based On CFS

Definition 4 [3]: A CFS A={([ψ-T1,ψ+T1],ξA)},B={([ψ-T2,ψ+T2],ξB)} is two CFNs and K>0 is any real number then we have

1. Ac={((1-ξA),[(1-ψ+TΓ),(1-ψ-TΓ])};
2. A⋎ B={([max(ψ-TΓ,ψ-TΓ),max(ψ+TΓ,ψ+TΓ)],min(ξA,ξB)};
3. A⋏ B={([min(ψ-TΓ,ψ-TΓ),min(ψ+TΓ,ψ+TΓ)],max(ξA,ξB)};
4. A⊕ B={([(ψ-T1+ψ-T2-ψ-T1ψ-T2),(ψ+T1+ψ+T2-ψ+T1ψ+T2)],(ξAξB))};
5. A⊗B={([(ψ-TΓψ-TΓ),(ψ+TΓψ+TΓ)],(ξA+ξB-ξAξB))};
6. K.A={[(1-(1-ψ-TΓ)K)],([(1-(1-ψ-TΓ)K)],ξAK)};
7. AK={[(ψ-TΓ)K],([((ψ-TΓ)K)],1-1-ξAK)};
8. A≤B, If ψ-TΓ⩽ ψ-TΓ, ψ+TΓ⩽ ψ+TΓ, ξA⩾ ξB;
9. A=B, If ψ-TΓ=ψ-TΓ, ψ+TΓ=ψ+TΓ,ξA=ξB.

Definition 5[3]: The accuracy and score functions for comparing two CFNs are defined as Let A={([ψ-TΓx,ψ+TΓx],ξA)}, be a (CFN, the score function of A)

SA=ψ-TΓ+ψ+TΓ-ξA3 (4)

and the accuracy function of A is

AA=ψ-TΓ+ψ+TΓ+ξA3 (5)

2.6. Cubic Fuzzy Weighted Averaging Operators (CFWAOs)

Definition 6 [3]: Let A={([ψ-TΓx,ψ+TΓx],ξA)} be a family of CFNs, ω=(ω1,ω2,ω3,…ωn)T be a weighted vector with ω∈[0,1] such that j=1nωj=1 then it is known as Cubic Fuzzy Weighted Averaging (CFWA) operators that is CFWAA1,A2,…,An=1-j=1n1-ψ-Ajωj,1-j=1n1-ψ+Ajωj,j=1n(ξAj)ωj (6)

Definition 7[3]: Let A={([ψ-TΓx,ψ+TΓx],ξA)} be a family of CFNs, ω=(ω1,ω2,ω3,…ωn)T be a weighted vector with ω∈[0,1] such that j=1nωj=1 then it is known as Cubic Fuzzy Weighted Geometric (CFWG) operators that are CFWGA1,A2,…,An=j=1n1-ψ-Ajωj,j=1n1-ψ+Ajωj,1-j=1n(1-ξAj)ωj (7)

2.7. Hesitant Fuzzy Set

Definition 8 [4]: Let X is nonempty set. A HFS is function that when we applied on X returns a finite subset of [0, 1]. A HFS is denoted and defined by, ϖ={x,ϱx:x∈X} (8)

Where is a set of various diverse values in [0, 1], which denotes the probable membership degrees of an element x∈X , for assistance we call ϱ = ϱx a HFE.

2.8. Operational Laws Based On HFS

ϱi=hvi=i=1,2 , be FNs:

1. ϱΓ1=h(v1)Γ;
2. Γ.ϱ1=h1-(1-v1)Γ;
3. ϱ1⊕ϱ2=hv1+v2-v1v2;
4. ϱ1⊕ϱ2=hv1v2;
5. ϱc1=h(1-v1);
6. ϱ1⋎ ϱ2={v∈ϱ1⋎ ϱ2v⩾ max(ϱ1,ϱ2}};
7. ϱ1⋏ ϱ2=v∈ϱ1⋏ ϱ2v⩾ minϱ1,ϱ2.

2.9. Hesitant Fuzzy Weighted Aggregation (HFWAOs) operators

Definition 9 [5] : Let ϱi1,2,…,n be a family of HFNs,ω=(ω1,ω2,ω3,…ωn)T be a weighted vector with ω∈[0,1] such that j=1nωj=1 then it is known as Hesitant FWAO that is HFWAϱ1,ϱ2,…,ϱn=1-j=1n1-ϱiωj (9)

Definition 10 [5]: Let ϱi1,2,…,n be a family of HFNs,ω=(ω1,ω2,ω3,…ωn)T be a weighted vector with ω∈[0,1] such that j=1nωj=1 then it is known as Hesitant FWGO that is HFWGϱ1,ϱ2,…,ϱn=j=1nϱiωj (10)

2.10. Interval valued Hesitant Fuzzy Set (IVHFS)

Definition 11 [9]: Let X be a non-empty set and S[0,1] denotes the set of all closed of [0,1]. An Interval valued HFS on X is defined and denoted by, p={xjg(xj):xj∈X} (11)

Wherever g:X⟶S[0,1] denote all possible interval valued belonging of xj in X and g(xj)=[g-,g+] , where g-=infg and g+=supg.

2.11. Operational Laws

Definition 12 [9]: Let p1,p2,p3 be any three IVHFNs and Γ>0 then,

1. pc=[1-g+,1-g-];
2. p1⊕p2={[g1-+g2--g1-g2-,g1++g2+-g1+g2+]};
3. p1⊗ p2={[g1-g2-,g1+g2+]};
4. Γ.p1={[1-(1-g1-)Γ,1-(1-g1+)Γ]};
5. p1Γ.= [(g1-)Γ,(g1+)Γ]}.

2.12. Interval valued Hesitant Fuzzy weighted Aggregation Operators (IVHFWAOs)

Definition 13[9]: Let ϱi1,2,…,n be a family of IVHFNs, ω=(ω1,ω2,ω3,…ωn)T be a weighted vector with ω∈[0,1] such that j=1nωj=1 then it is known as interval valued hesitant fuzzy weighted averaging (IVHFWA) operators that is IVHFWAϱ1,ϱ2,…,ϱn=1-j=1n1-gj-ωj,1-j=1n1-gj+ωj (12)

Definition 14[9]: Let ϱi1,2,…,n be a family of HFNs,ω=(ω1,ω2,ω3,…ωn)T be a weighted vector with ω∈[0,1] such that j=1nωj=1 then it is known as Interval valued HFWGO that is IVHFWGϱ1,ϱ2,…,ϱn=j=1ngj-ωj,j=1ngj+ωj

2.13. Cubic Hesitant Fuzzy Set (CHFS)

Definition 15 [10]: Let X be the nonempty universal set. Then the set

ω={(x,pω=[p-ω,p+ω],θω(x))/x∈X} (14)

is said to be a CHFS of X, where pωx be an IVHFS in X and θω(x) is a HFS in X where pω=[p-ω,p+ω]. Some basic concept concerning to CS are defined in the following steps:

2.14. Operational Laws

i. **Definition:** A CS ω={(x,pω=[p-ω,p+ω],θω(x))/x∈X} is termed as an internal CHFS if the term p-ωx≤θω(x)≤p-ωx ;

ii. **Definition:** A CS ω={(x,pω=[p-ω,p+ω],θω(x))/x∈X} is termed as an external CHFS if the term θω(x)∉ (p-ω,p+ω)

1. ωc={(1-p-ω),1-(p+ω)],(1-θω)};
2. A⊕ B={([p-ω+p-ω-p-ωp-ω),p+ω+p+ω-p+ωp+ω)],(θωθω)};
3. A⊗B={([p-ωp-ω,p+ωp+ω],(θωθω)};
4. K.A={[(1-(1-p-ω)K,([(1-(1-p+ω)K)],(θω)K};
5. AK={[((p-ω)K,([((p+ω)K)],1-(1-θω)K};
6. A≤B, if p-ω⩽ p-ω,p+ω⩽ p+ω,θω⩾ θω;
7. A=B, if p-ω=p-ω,p+ω=p+ω,θω=θω.

2.15. Cubic Hesitant Fuzzy Weighted Aggregation Operators

Definition16 [10]: (CHFWA Operator) Consider chk(k=1,2,…,n) is group of Cubic HFEs. The Cubic HFSs is explained by the mapping Gn⟶G such that CHFWAch1,ch2,…,chn=⊕Rk=1nwkchk, Here w=w1,w2,…,wnT is weight direction of CHFEs by wK∈0,1 and k=1nwK=1. In different case if w=(1n,1n,…,1n)Tthen CHFWA operator develops as, CHFWA ch1,ch2,…,chn=⊕Rk=1n1nchk=1-k=1n1-μik-wK,1-k=1n1-μik+wK,k=1nρikwk; (15)

Definition 17[10]: CHFWG 46Operator Consider chkk=1,2,…,n is group of Cubic HFEs. The Cubic HFSs is defined by the mapping Gn⟶G such that CHFWGch1,ch2,…,chn=⊕Rk=1nwkchk, here w=w1,w2,…,wnT is weight direction of CHFEs by wK∈0,1 andk=1nwK=1. In different case if w=1n,1n,…,1nTthen the CHFWA operator develops as, CHFWG ch1,ch2,…,chn=⊕k=1n1nchk=k=1nμik-wK,k=1nμik+wK,1-k=1n1-ρikwk; (16)

Definition 18[10]: (R-CHFWG Operator) Assume Consider chk(k=1,2,…,n) is a collection of CHFEs. The R-weighted geometric operator for CHFSs (for short, R- CHFWG) is defined by the drawing Gn⟶G such that, R-CHFWGch1,ch2,…,chn=⊗ Rk=1nchkwk, here w=w1,w2,…,wnT is a weight direction of CHFEs by wK∈0,1 and k=1nwK=1. In special case if w=(1n,1n,…,1n)Tthen the R-CHFWG operator becomes as, R-CHFWG ch1,ch2,…,chn=⊗ Rk=1nchk1n (17)

2.16. Soft Set (SS)

Definition 19 [12]: Let X is a nonempty set with attributes of a set Ā denoted like:

FA= e, Fe e∈Ā , F e∈PŪ (18) From Ā to P Ū is corresponding with F i.e. F: Ā→ P Ū.

Example 3: Let Ū=ô1,ô2,ô3,ô4denote the set of bikes under consideration and E= {black, low fuel consumptions, mechanical fit} is attributes set, a SS is expressed as:

F e1=ô1,ô2, F e2=ô2,ô3, F e3=ô1,ô2,ô3,

F, Ā=F (e1), F (e2), F (e3).

2.17. Fuzzy Soft Set (FSS)

Definition 20 [13]: A Fuzzy SS over Ū with attributes of set expressed like: Fṁ= e, Fe e∈Ā , Fe∈FPŪ (19)

From Ā to FP(Ū) is corresponding with F. i.e. F: Ā→FP(Ū).

Example 4: Example 2.11.1, let's take and usage the subsequent Fuzzy Ss for a Fuzzy SS results:

F (e1) =(ô1,0.5),(ô2,0.8), F (e2)=(ô2,0.6),(ô3,0.3), F (e3)=(ô1,0.3),(ô2,0.7),(ô3,0.4),

The FSS finished the set of attributes Ā is expressed as

Fṁ=F (e1), F (e2), F (e3).

2.18. Hesitant Fuzzy Soft Set (HFSS)

Definition 21[14]: Let X be a universe and let A be set of attributes and also let HFP(X) be set which consist all Hesitant Fuzzy subsets of X. A pair (F, A) is known as Hesitant FSS on X, where F is a mapping from A to HFP(X) denoted by

F:A→ HFPX. (20)

2.19. Hesitant Fuzzy Soft Aggregation Operators (HFSAOs)

Definition 22 [14]: Let Ki1,2,…,n be a family of HFSNs,ω=(ω1,ω2,ω3,…ωn)T be a weighted vector with ω∈[0,1] such that j=1nωj=1 then it is known as hesitant fuzzy soft weighted averaging (HFSWA) operators that are:

HFSWA(K1,K2,…,Kn)= Kij=1n1-ϱiωj; (21)

Definition 23[14]: Let ϱi1,2,…,n be a family of HFNs,ω=(ω1,ω2,ω3,…ωn)T be a weighted vector with ω∈[0,1] such that j=1nωj=1 then it is known as hesitant fuzzy weighted geometric (HFSWG) operators that is

HFSWG(K1,K2,…,Kn)= Kij=1nϱiωj; (22)

2.20. Interval Valued Hesitant Fuzzy Soft Set

Definition 24 [14]: Let X be a non-empty set and S[0,1] denotes the set of all closed of [0,1]. An Interval valued HFS on X is denoted and defined by, p={e,xjg(xj):xj∈X} (23)

where g:X→S[0,1] denote all possible interval valued membership of xj in X and =[g-,g+] , where g-=infg and g+=supg.

2.21. Interval valued Hesitant Fuzzy Soft Aggregation operators (IVHFSAOs)

Definition 25 [14]: Let ϱi1,2,…,n be a family of IVHSFNs,ω=(ω1,ω2,ω3,…ωn)T be a weighted vector with ω∈[0,1] such that j=1nωj=1 then it is known as Interval Valued HFWA operators that are:

IVHFSWA(ϱ1,ϱ2,…,ϱn)=1-j=1n1-gj-ωj,1-j=1n1-gj+ωj. (24)

Definition 26[14]: Let ϱi1,2,…,n be a family of IVHSFNs,ω=(ω1,ω2,ω3,…ωn)T be a weighted vector with ω∈[0,1] such that j=1nωj=1 then it is known as Interval Valued HFWG operators that are:

IVHFSWG(ϱ1,ϱ2,…,ϱn) =j=1ngj-ωj,j=1ngj+ωj (25)

CUBIC HESITANT FUZZY SOFT SET AND ITS APPLICATION TO MULTI-CRITERIA DECISION-MAKING

We employ operations laws on Cubic HFSSs in this chapter and recommend for new aggregation operators for instance, Cubic HFSWA operators, Cubic HFSOWA operators, Cubic HFSWA operators and Cubic HFSOWG operators and their properties as well as MCDM algorithms.

2.22. Proposed Laws of CHFSS

Definition 27: Let X be a non-empty set. The CHFSS on X is defined by,

α=h,ξh,Hhh∈X (26)

Where ξan (IVHFSS) in X and H is a (HFSS) in X. A CHFSS is simply denoted as α=ξ,H.

Example 5: Suppose that U={P1,P2, P3} is the set of software development projects under consideration, A={e1,e2,e3} is the set of attributes, e_1 is for economic feasibility, e_2 is for technological feasibility, and e_3 is for staff feasibility. A CHFSS is displayed as follows:

F(e1)=P1,0.1,0.3,0.4,0.5,0.2,0.4,(P2,{[0.3,0.4],[0.1, 0.3]},{0.1,0.5}),(P3,{[0.2,0.5],[0.1,0.7]},{0.1,0.7})};

F(e2)=P1,0.2,0.4,0.1,0.3,0.2,0.7,(P2,{[0.1,0.3],[0.2, 0.4]},{0.5,0.6}),(P3,{[0.2,0.7],[0.2,0.8]},{0.2,0.5})};

F(e3)=P1,0.1,0.3,0.4,0.5,0.2,0.4,(P2,{[0.3,0.4],[0.1, 0.3]},{0.1,0.5}),(P3,{[0.2,0.5],[0.1,0.7]},{0.1,0.7})}.

Definition 28: Let X is a non-empty set, A cubic hesitant fuzzy soft set α=ξ,H is assumed to be an Internal Cubic Hesitant Fuzzy Soft Set (ICHFSS) if Hi(x)∈[ξi-(x),ξi+x],∀ x∈X .

Example 6: The example provided below is an ICHFS,

F(e1)=P_1,0.1,0.3,0.4,0.5,0.2,0.6,(P_2,{[0.3,0.6],[0.1, 0.3]},{0.4,0.2}),(P_3,{[0.2,0.5],[0.1,0.7]},{0.3,0.5})},

F(e2)=P_1,0.2,0.4,0.1,0.3,0.3,0.2,(P_2,{[0.1,0.5],[0.2, 0.6]},{0.4,0.5}),(P_3,{[0.2,0.7],[0.2,0.8]},{0.3,0.5})},

F(e3)=P_1,0.1,0.3,0.4,0.7,0.2,0.5,(P_2,{[0.3,0.4],[0.1, 0.7]},{0.2,0.5}),(P_3,{[0.2,0.5],[0.1,0.7]},{0.2,0.5})}

α={Fe1,Fe2,F(e3)}.

Definition 29: X is a nonempty set, A Cubic hesitant set α=ξ,H.is known to be an External CS if Hi(x) ∉ [ξi-(x),ξi+x],∀ ,∀ x∈X.

Theorem: Suppose α=ξ,H be a CHFSS on X that is not the ECHFSS. Then there exist x∈X such that Hi(x) ∈[ξi-(x),ξi+x],∀ ,∀ x∈X.

Definition (Equality) 30: Let α=ξ,H, and ϑ=σ,Q be any two CHFSs on X and we denote α=ϑ iff,

i. ξix=σix∀i∈∧ (Where ∧ is an index) and∀ x∈X, ξix=[ξi-(x),ξi+x],∀ ∈ C(x), σi=[σi-x,σi+x]∈ϑ (x) are the membership grade intervals of the elementx∈X.

ii. Hx=Q(x).

Definition (P-Order) 31: Letα=ξ,H, and ϑ=σ,Q be any two CHFSs on X and we denote α=ϑ iff,

i. ξix≤σix∀i∈∧ (Where ∧ is an index) and, ∀ x∈X, ξix=[ξi-(x),ξi+x],∀ ∈[C(x), σi=[σi-x,σi+x] ∈ ϑ (x) are the membership grade intervals of the elementx∈X.

ii. Hx≤Q(x).

Definition (R-Order) 32: Letα=ξ,H, and ϑ=σ,Q be any two CHFSs on X and we denote α=ϑ iff,

i. ξix≥σix∀i∈∧ (Where ∧ is an index) and ∀ x∈X,ξix=[ξi-(x),ξi+x],∀ ∈ C(x), σi=[σi-x,σi+x] ∈ ϑ (x) are the membership grade intervals of the elementx∈X.

ii. Hx≥Q(x).

Example (P-Order) 7: Letα=ξ,H, and ϑ=σ,Q be any two CHFSs on X defined as follows,

F(e1)=P1,0.1,0.3,0.4,0.5,0.2,0.6,(P2,{[0.3,0.6],[0.1, 0.3]},{0.4,0.2}),(P3,{[0.2,0.5],[0.1,0.7]},{0.3,0.5})},

F(e2)=P1,0.2,0.4,0.1,0.3,0.3,0.2,(P2,{[0.1,0.5],[0.2, 0.6]},{0.4,0.5}),(P3,{[0.2,0.7],[0.2,0.8]},{0.3,0.5})},

F(e3)=P1,0.1,0.3,0.4,0.7,0.2,0.5,(P2,{[0.3,0.4],[0.1, 0.7]},{0.2,0.5}),(P3,{[0.2,0.5],[0.1,0.7]},{0.2,0.5})}

α={Fe1,Fe2,F(e3)}.

F(e1)=P1,0.1,0.5,0.4,0.6,0.3,0.7,(P2,{[0.3,0.7],[0.1, 0.5]},{0.1,0.2,0.3,0.4}),(P3,{[0.2,0.6],[0.1,0.9]},{0.1, 0.3,0.5})},

F(e2)=P1,0.2,0.6,0.1,0.5,0.1,0.3,0.2,(P2,{[0.1,0.7], [0.2,0.9]},{0.4,0.6}),(P3,{[0.2,0.9],[0.2,0.9]},{0.1,0.3, 0.5})},

F(e3)=P_1,0.1,0.5,0.2,0.7,0.1,0.2,0.5,(P_2,{[0.1,0.5],[0.1,0.9]},{0.1,0.2,0.5}),(P_3,{[0.1,0.8],[0.1,0.9]},{0.1, 0.2,0.5})} ϑ={Pe1,Pe2,P(e3)}.

Example (P-Order) 8: Let α=ξ,H, and ϑ=σ,Q be any two CHFSs on X defined as follows: F(e1)=P_1,0.1,0.5,0.4,0.6,0.3,0.7,(P_2,{[0.3,0.7],[0.1, 0.5]},{0.1,0.2,0.3,0.4}),(P_3,{[0.2,0.6],[0.1,0.9]},{0.1, 0.3,0.5})},

F(e2)=P_1,0.2,0.6,0.1,0.5,0.1,0.3,0.2,(P_2,{[0.1,0.7],[0.2,0.9]},{0.4,0.6}),(P_3,{[0.2,0.9],[0.2,0.9]},{0.1,0.3, 0.5})},

F(e3)=P_1,0.1,0.5,0.2,0.7,0.1,0.2,0.5,(P_2,{[0.1,0.5],[0.1,0.9]},{0.1,0.2,0.5}),(P_3,{[0.1,0.8],[0.1,0.9]},{0.1, 0.2,0.5})}

2.23. Operation on Cubic Hesitant Fuzzy Soft Set

Let X is a non-empty set. α1=h,ξα1h,Hihh∈X and α2=h,ξα2h,Hihh∈X be any two CHFSNs on X then where ξα1=ξlα1,ξuα1,ξα2=ξlα2,ξuα2 ,Hi={ϑi}

i. α1⊕α2={[ξlα1+ξlα2-ξlα1ξlα2,ξuα1+ξuα2-ξuα1ξuα2],(ϑ1ϑ2)};

ii. α1⊗α2={([ξlα1ξlα2,ξuα1ξuα2)],(ϑ1+ϑ2-ϑ1ϑ2)};

iii. αc1={([1-ξuα1,1-ξlα1)],(1-ϑ1)};

iv. K.α1={[(1-(1-ξlα1)K,(1-(1-ξuα1)K],(ϑ1)K};

v α1K={[ξlα1K,ξuα1K],1-(1-ϑ1)K};

Example 9: Let's consider α1=0.1,0.4,0.2,0.5,{0.1,0.2},α2=0.2,0.5,0.1,0.7,{0.2,0.3} and K=3 then we have

i. α1⊕α2=0.1+0.2-0.1.0.2,0.4+0.5+-0.4.0.5,0.2+0.1-0.2.0.1,0.5+0.7+-0.5.0.7,0.1.0.2.0.2.0.3=0.28,0.70,0.28,0.85,0.02,0.06;

ii. α1⊗α2=0.1.0.2,0.4.0.5,0.2.0.1,0.5.0.7,0.1+0.2-0.1.0.2,0.2+0.3-0.2.0.3=0.02,0.2,0.02,0.35,0.28,0.44;

iii. αc1=1-0.4,1-0.1,1-0.5,1-0.2,1-0.1,1-0.2=0.6,0.9,0.5,0.8,0.9,0.8;

iv. 3.α1=(1-(1-0.1)3,(1-(1-0.4)3,(1-(1-0.2)3,(1-(1-0.5)3,0.13,0.23=0.271,0.784,0.488,0.875,0.001,0.008;

v. α13=[0.13,0.43,[0.23,0.53],1-(1-0.1)3,1-(1-0.2)3}=[0.001,0.064,[0.008,0.125],0.271,0.488};

2.24. Score and Accuracy function of CHFSS

Definition 33: Let X is a non-empty set and αi=h,ξαih,Hihh∈X be a CHFSNs on X then the score of αi is denoted and defined by

Λαi=1⋆ αi(ξlαi+ξuαi-⋆ αi2+ϑi) (27)

where ⋆ αi is the number of elements in αi, ξα1=ξlαi,ξuαi ,Hi={ϑi}.

αi=h,ξαih,Hihh∈Xi=1,2 , represent two CHFSNs. Then, according to the definition of Λ and a, we have

i. Λα1⩾ Λα2⇔α1⩾ α2;

ii. Λα1⩽ Λα2⇔ α1⩽ α2;

iii. Λα1=Λα2 ⇔α1=α2;

- Γ(α1)⩾ Γ(α2)⇔α1⩾ α2;
- Γα1⩽ Γα2⇔α1⩽ α2;
- Γα1=Γα2⇔α1=α2;

Theorem 4: For two CHFSNs, α1and α2; we have to prove that

i. α1 ⊕ α2=α2 ⊕ α1;

ii. α1⊗ α2=α1⊗ α2.

Theorem 5: For any three CHFSNs, α1,α2 and α3 we have to prove that

i. α1⊕(α2⊕α2)=(α1⊕α2)⊕α3;

ii. α1⊗ (α2⊗ α3)=(α1⊗ α2)⊗ α3.

Theorem 6: For any two CHFSNs αi,i=1,2, the real numbers k1, k2,k3>0,

i. kα1⊕α2=kα1⊕ kα2;

ii. α1⊗α2k=(α1)k⊗(α2)k;

iii. k1α1⊕ k2α1=k1+k2α1;

iv. α1k1⊗α1k2=(α1)k1+k2;

Proof: We have to prove (1) and (3), though another may be as considered. α1=ξ1,H1 and α2=ξ2,H2

i. α1⊕α2=ξlα1+ξlα2-ξlα1ξlα2),ξuα1+ξuα2-ξuα1ξuα2),(ϑ1ϑ2);

Also k.α1=(1-(1-ξlα1)K,((1-(1-ξuα1)K),(ϑ1)K. Now

K.α1⊕α2=(1-(1-([ξlα1+ξlα2-ξlα1ξlα2))K,((1-(1-ξuα1+ξuα2-ξuα1ξuα2)K)],ϑ1ϑ2K= (1-(1-ξlα1)K,((1-(1-ξuα1)K),(ϑ1)K⊕(1-(1-ξlα1)K,((1-(1-ξuα1)K),(ϑ1)K =kα1⊕ kα2

ii. α1⊗α2=ξlα1ξlα2,ξuα1ξuα2,(ϑ1ϑ2) and α1K=[(ξlα1)K,(ξuα1)K,1-(1-ϑ1)K. α1⊗α2K=(ξlα1ξlα2)K,(ξuα1ξuα2)K,1-1-ϑ1ϑ2K={[((ξlα1)K,([((ξuα1)K)],1-(1-ϑ1)K}⊗{[((ξlα2)K,([((ξuα2)K)],1-(1-ϑ2)K}=(α1)K⊗(α2)K.

iii. K1α1⊕ K2α1=K1+K2α1
α1⊕α2={([ξlα1+ξlα2-ξlα1ξlα2),ξuα1+ξuα2-ξuα1ξuα2)],(ϑ1ϑ2)} and K.α1={[(1-(1-ξlα1)K,([(1-(1-ξuα1)K)],(ϑ1)K}.
K1α1⊕ K2α1={[(1-(1-ξlα1)K1,([(1-(1-ξuα1)K1)],ϑ1K1}⊕{[(1-(1-ξlα1)K2,([(1-(1-ξuα1)K2)],ϑ1K2}={[(1-(1-ξlα1)K1+K2,([(1-(1-ξuα1)K1+K2)],ϑ1K1+K2}=K1+K2α1.

iv. α1K1⊗α1K2=(α1)K1+K2
α1K1={[((ξlα1)K1,([((ξuα1)K1)],1-(1-ϑ1)K1} and α1K2={[((ξlα1)K2,([((ξuα1)K2)],1-(1-ϑ1)K2}.
α1K1⊗α1K2={[((ξlα1)K1,([((ξuα1)K1)],1-1-ϑ1K1}⊗={[((ξlα1)K2,([((ξuα1)K2)],1-1-ϑ1K2}={[((ξlα1)K1+K2,([((ξuα1)K1+K2)],1-(1-ϑ1)K1+K2}=(α1)K1+K2

2.25. Operators of Cubic Hesitant Fuzzy Soft WA Aggregation

In the following unit, we introduce arithmetic AOs using CHFSNs grounded algebraic procedures. These operators, namely the Cubic HFSWA operator, and Cubic HFSOWA operator, will be considered in detail to discover their major properties.

Definition 34: Suppose αij=ξij,Hij be a group of CHFSNs. Then cubic hesitant fuzzy soft weighted average is a mapping CHFSWA :αijn→αij, with ωi>0 and i=1nωi=1 where ωi is a weighted vector.

Definition 35: Suppose αij=ξij,Hij,wherei=1,2,..,m and j=1,2,...,n, represents a group of CHFSNs, the cubic hesitant fuzzy soft weighted average (CHFSWA) operators as a mapping CHFSWA :αijn→αijcharacterized by:
CHFSWAα11,α12,...,αij=⊕j=1nΔj⊕i=1m∇i αij=1-j=1ni=1m1-ξijl∇iΔj,1-j=1ni=1m1-ξiju∇iΔj,j=1ni=1mHij∇iΔj (28)
Where ∇i={∇1,∇2,...,∇i}T,Δj={Δ1,Δ2,..,Δj}T, represent a weighted vector of ∪ij respectively, such that Δj> 0,∇i> 0 and j=1nΔj=1,i=1m∇i=1.

Theorem 8: Suppose αij=ξij,Hij,wherei=1,2,..,m and j=1,2,...,n, represents a group of CHFSNs and ∇i={∇1,∇2,...,∇i}T,Δj={Δ1,Δ2,..,Δj}T weighted vector satisfied the condition Δj> 0,∇i> 0 and j=1nΔj=1,i=1m∇i=1. Then the aggregated values obtained by the CHFSWA operator are also CHFSNs.

Proof: we prove with the help of mathematical induction for n=2=m.

(i) CHFSWA(α11,α12)=1-[1-g11-∇1.1-g21-∇2Δ1.1-g12-∇1.1-g22-∇2Δ2,1-g11+∇1.1-g21+∇2Δ1.1-g12+∇1.1-g22+∇2Δ2],H11∇1.H21∇2Δ1.H12∇1.H22∇2Δ2. Now let us consider the result is true for n=k=m

(ii) CHFSWA(α11,α12,α13,...,αkk)=[1-j=1ki=1k1-gij-∇iΔj,1-j=1ki=1k1-gij+∇iΔj],j=1ki=1kHij∇iΔj.Now the result is true for n= k+1=m

(iii) CHFSWAα11,α12,α13,...,αk+1k+1=1-j=1k+1i=1k+11-gij-∇iΔj,1-j=1k+1i=1k+11-gij+∇iΔj,j=1k+1i=1kHij∇iΔj+1-j=1k+1i=1k+11-gij-∇iΔj,1-j=1k+1i=1k+11-gij+∇iΔj,j=1k+1i=1kHij∇iΔj

Example 10:α1=0.1,0.4,0.2,0.5,{0.1,0.2},α2=0.2,0.5,0.1,0.7,{0.2,0.3} and α3=0.3,0.6,0.4,0.9,{0.3,0.4} weight

vectors Δi={0.20,0.30,0.50} and ∇j={0.19,0.31,0.50}.
CHFSWAα11,α12,…,αij=⊕j=1nΔj⊕i=1m∇i αij=1-j=1ni=1m1-ξijl∇iΔj,1-j=1ni=1m1-ξiju∇iΔj,j=1ni=1mHij∇iΔj=1-1-0.10.200.19(1-0.20.30)0.31(1-0.30.50)0.501-1-0.40.200.19(1-0.50.30)0.31(1-0.60.50)0.50,,1-1-0.20.200.19(1-0.10.30)0.31(1-0.40.50)0.501-1-0.50.200.19(1-0.70.30)0.31(1-0.90.50)0.50,({0.1,0.2}0.20)0.19,({0.2,0.3}0.20)0.19({0.3,0.4}0.20)0.19=0.0978,0.2687,0.1358,0.51030.4793,0.3788,0.2624;

2.26. Properties of CHFSWA Operators

1. **(Idem potency)** We introduce sets of CHFSNs represented as αij=ξij,Hij,wherei=1,2,..,m and j=1,2,…,n. Shows each element in this set have the same values, i.e. αij=αij. Then, applying the Cubic HFSWA to α11,α12,…,αij outcomes in α. i.e. CHFSWAα11,α12,…,αij=α. (29)

Proof: Since αij=ξij,Hij,where i=1,2,..,m and j=1,2,…,n., then by using equation (9) we have, CHFSWA(α11,α12,α13,…,αmn)=1-j=1ni=1m1-gij-∇iΔj,1-j=1ni=1m1-gij+∇iΔj,j=1ni=1mHij∇iΔj=1-1-gij-∇iΔj,1-1-gij+∇iΔj,Hij∇iΔj=α

Likely, it could be proved many characteristics of the Fuzzy BPSHWA operator given are is under.

1. **(Boundedness)** Let αij=ξij,Hij,where i=1,2,..,m and j=1,2,…,n., be a group of CHFSNs, and α-ij=minijαij , α+ij=maxijαij , then α-ij⩽ CHFSWA(α11,α12,α13,…,αmn)⩽ α+ij; (30)

2. **(Monotonicity)** Suppose αij=ξij,Hij and αij=(ξij,Hij) be a two groups of CHFSNs, if αij⩽ αij .Then CHFSWA(α11,α12,α13,…,αmn)⩽ CHFSWAα11,α12,…,αij. (31)

3. **(Commutatively Property)** Suppose αij=ξij,Hij and αij=(ξij,Hij) be a two groups of CHFSNs, if αij⩽ αij .Then CHFSWA(α11,α12,α13,…,αmn)⩽ CHFSWAα11,α12,…,αij. (32)

Definition 36: Suppose αij=ξij,Hij,wherei=1,2,..,m and j=1,2,…,n, represents a group of CHFSNs, the cubic hesitant fuzzy soft ordered weighted average (CHFSOWA) operators as a mapping CHFSOWA :αϖ(ij)n→αijcharacterized by: CHFSOWAαϖ(11),αϖ(12),…,αϖ(ij)=⊕j=1nΔj⊕i=1m∇iαϖ(ij)=1-j=1ni=1m1-ξϖijl∇iΔj,1-j=1ni=1m1-ξϖiju∇iΔj,j=1ni=1mHϖ(ij)∇iΔj (33)

Where ∇i={∇1,∇2,…,∇i}T,Δj={Δ1,Δ2,..,Δj}T, represent a weighted vector of υij respectively, such that Δj> 0,∇i> 0 and j=1nΔj=1,i=1m∇i=1.

Theorem 3: Suppose αij=ξij,Hij,wherei=1,2,..,m and j=1,2,…,n, represents a group of CHFSNs and ∇i={∇1,∇2,…,∇i}T,Δj={Δ1,Δ2,..,Δj}T weighted vector satisfied the condition Δj> 0,∇i> 0 and j=1nΔj=1,i=1m∇i=1. Then the aggregated values obtained by the CHFSOWA operator are also CHFSNs.

The features of CHFSOWA operator listed lower can be straightforwardly established.

1. **(Idempotency Property)** Suppose αij=ξij,Hij, where i=1,2,..,m and j=1,2,…,n, represent set of CHFSNs such that αij=α for all i,j. Then, it follows that CHFSOWAαϖ(11),αϖ(12),…,αϖ(ij)=α. (34)

2. **(Boundedness)** Suppose αij=ξij,Hij,where i=1,2,..,m and j=1,2,…,n be a group of CHFSNs, and α-ij=minijαij , α+ij=maxijαij , then α-ij⩽ CHFSOWAαϖ(11),αϖ(12),…,αϖ(ij)⩽ α+ij. (35)

3. **(Monotonicity)** Suppose αij=ξij,Hij and αij=(ξij,Hij) be a two groups of CHFSNs, if αij⩽ αij.Then,
CHFSOWAαϖ(11),αϖ(12),…,αϖ(ij)⩽
CHFSOWAαϖ(11),αϖ(12),…,αϖ(11) (36)

4. **(Commutatively Property)** Suppose αij=ξij,Hij and αij=(ξij,Hij) be two groups of CHFSNs, if αij⩽ αij. Then,
CHFSOWAαϖ(11),αϖ(12),…,αϖ(ij)⩽
CHFSOWAαϖ(11),αϖ(12),…,αϖ(11).(37)

2.27. Cubic Hesitant Fuzzy Soft Weighted Geometric Aggregation (CHFSWG) operators

In this segment, we present geometric aggregation operators using CHFSNs based algebraic operations. These operators, namely the Cubic HFSWG operator, and the Cubic HFSOWG operator would then be studied with due operation to discover its important aspects.

Description 37: Let αij=ξij,Hij,wherei=1,2,..,m and j=1,2,…,n, represents a group of CHFSNs, the cubic hesitant fuzzy soft weighted geometric (CHFSWG) operators as a mapping CHFSWG :αijn→αijcharacterized by:
CHFSWGα11,α12,…,αij=⊗j=1nΔj⊗i=1m∇i αij=j=1ni=1mξijl∇iΔj,j=1ni=1mξiju∇iΔj,1-j=1ni=1m1-Hij∇iΔj(38)

Where ∇i={∇1,∇2,…,∇i}T,Δj={Δ1,Δ2,..,Δj}T, represent a weighted vector of υij respectively, such that Δj> 0,∇i> 0 and j=1nΔj=1,i=1m∇i=1.

Theorem 8: Suppose αij=ξij,Hij,wherei=1,2,..,m and j=1,2,…,n, represents a group of CHFSNs and ∇i={∇1,∇2,…,∇i}T,Δj={Δ1,Δ2,..,Δj}T weighted vector satisfied the condition Δj> 0,∇i> 0 and j=1nΔj=1,i=1m∇i=1. Then the aggregated values obtained by the CHFSWG operator are also CHFSNs.

Example 9:α1=0.1,0.4,0.2,0.5,{0.1,0.2},α2=0.2,0.5,0.1,0.7,{0.2,0.3} and α3=0.3,0.6,0.4,0.9,{0.3,0.4} weight vectors Δi={0.20,0.30,0.50} and ∇j={0.19,0.31,0.50}.
CHFSWGα11,α12,…,αij=⊗j=1nΔj⊗i=1m∇i αij=j=1ni=1mξijl∇iΔj,j=1ni=1mξiju∇iΔj,1-j=1ni=1m1-Hij∇iΔj=0.10.200.19(0.20.30)0.31(0.30.50)0.50 0.40.200.19(0.50.30)0.31(0.60.50)0.50,,0.20.200.19(0.10.30)0.31(0.40.50)0.500.50.200.19(0.70.30)0.31(0.90.50)0.50,(1-1-{0.1,0.2}0.20)0.19,(1-1-{0.2,0.3}0.20)0.19(1-1-{0.3,0.4}0.20)0.19=
0.4521,0.3316,0.4316,0.5543

2.28. Properties of CHFSWG

1. **(Idem potency)** Now contemplate a pool of CHFSNs indicated αij=ξij,Hij, where i=1,2,,..,m and j=1,2,…,n. Simulating entire features in that pool equivalent, i.e.,αij=α. At that point, operating Cubic CHFSWG to α11,α12,…,αijupshots in α. i.e. CHFSWGα11,α12,…,αij=α. (39)

2. **(Boundedness)** Suppose αij=ξij,Hij,where i=1,2,,..,m and j=1,2,…,n be a group of CHFSNs, and α-ij=minijαij , α+ij=maxijαij , then α-ij⩽ CHFSWGα11,α12,…,υij⩽ α+ij. (40)

3. **(Monotonicity)** Suppose αij=ξij,Hij and αij=(ξij,Hij) be a two groups of CHFSNs, αij⩽ αij.Then, CHFSWGα11,α12,…,αij⩽ CHFSWGα11,α12,…,αij. (41)

4. **(Commutatively Property)** Suppose αij=ξij,Hij and αij=(ξij,Hij) be a two groups of CHFSNs, αij⩽ αij.Then,
CHFSWGα11,α12,…,αij⩽
CHFSWGα11,α12,…,αij.

Definition 38: Suppose αij=ξij,Hij,where i=1,2,..,m and j=1,2,…,n, represents a group of CHFSNs, the cubic hesitant fuzzy soft ordered weighted geometric (CHFSOWG) operators as a mapping CHFSOWG: αijn→

αij characterized by: CHFSOWGαϖ(11),αϖ(12),...,αϖ(ij)=⊗j=1n∆j⊗i=1m∇iαϖ(ij)=j=1ni=1mξϖ(ij)l∇i∆j,j=1ni=1mξϖ(ij)u∇i∆j,1-j=1ni=1m1-Hϖ(ij)∇i∆j (42)

Where ∇i={∇1,∇2,...,∇i}T,∆j={∆1,∆2,..,∆j}T, represent a weighted vector of ʊij respectively, such that ∆j> 0,∇i> 0 and j=1n∆j=1,i=1m∇i=1.

Theorem 5: Suppose αij=ξij,Hij,wherei=1,2,..,m and j=1,2,...,n, represents a group of FBPSNs and ∇i={∇1,∇2,...,∇i}T,∆j={∆1,∆2,..,∆j}T weighted vector satisfied the condition ∆j> 0,∇i> 0 and j=1n∆j=1,i=1m∇i=1. Then the aggregated values obtained by the CHFSOWG operator are also CHFSNs.

Example 9:

α1=0.1,0.4,0.2,0.5,{0.1,0.2},α2=0.2,0.5,0.1,0.7,{0.2,0.3} and α3=0.3,0.6,0.4,0.9,{0.3,0.4} weight vectors ∆i={0.20,0.30,0.50} and∇j={0.19,0.31,0.50}.

CHFSWAα11,α12,...,αij=⊕j=1n∆j⊕i=1m∇i αij=j=1ni=1mξijl∇i∆j,j=1ni=1mξiju∇i∆j,1-j=1ni=1m1-Hij∇i∆j=0.10.200.19(0.20.30)0.31(0.30.50)0.50 0.40.200.19(0.50.30)0.31(0.60.50)0.50,,0.20.200.19(0.10.30)0.31(0.40.50)0.500.50.200.19(0.70.30) 0.31(0.90.50)0.50,(1-1-{0.1,0.2}0.20)0.19,(1-1-{0.2,0.3}0.20)0.19(1-1-{0.3,0.4}0.20)0.19

Characteristics of CHFSWG drivers labeled could candidly validated.

1. **(Idem potency)** Consider a throng of CHFSNs stand for αij=ξij,Hij, where i=1,2,,..,m and j=1,2,...,n. Take up all essentials in that assemblage is the same, i.e., αij=α. Now, employing Cubic HFSWG to αϖ(11),αϖ(12),...,αϖ(ij) concludes in αϖ. i.e. CHFSWGαϖ(11),αϖ(12),...,αϖ(ij)=αϖ. (43)
2. **(Boundedness)** Suppose αij=ξij,Hij,where i=1,2,,..,m and j=1,2,...,n be a group of CHFSNs, and α-ij=minijαij , α+ij=maxijαij , then α-ij⩽ CHFSWGα11,α12,...,ʊij⩽ α+ij. (44)
3. **(Monotonicity)** Suppose αij=ξij,Hij and αij=(ξij,Hij) be a two groups of CHFSNs, αij⩽ αij.Then, CHFSWGα11,α12,...,αij⩽ CHFSWGα11,α12,...,αij. (45)
4. **(Commutatively Property)** Suppose αij=ξij,Hij and αij=(ξij,Hij) be a two groups of CHFSNs, αij⩽ αij.Then, CHFSWGα11,α12,...,αij⩽ CHFSWGα11,α12,...,αij. (46)

2.29. Novel MCDM Based on Proposed Aggregation Operators

In this thesis area, we employ the technique of MCDM by Cubic HFSWA and Cubic HFSWG operators to cope MCDM conditions, where Cubic Hesitant Fuzzy Soft Numbers (CHFSNs) assist as criteria weightings and values. Firstly, we deliver a mathematical structure for MCDM scenarios integrating data of cubic hesitant fuzzy soft set. To demonstrate the relevancy of our planned methods, we introduce a descriptive example comprising a cooperative of agriculturalists deciding which crop to plant on a huge shared plot of land. The decision must reflect numerous criteria, such as Profit Potential, Water Requirements, Growth Duration, and Market Demand, and collect input from diverse experts, including an Agronomist, an Economist, a Hydrologist, and a Market Analyst. The substitutes under consideration contain Wheat, Corn, Soybeans, and Rice. Let P = {Wheat, Corn, Soybeans, Rice} represent a separate set of substitutes, and E={E1,E2,....,En} represent the set of exports. Let

∇i={∇1,∇2,...,∇i}T,Δj={Δ1,Δ2,..,Δj}T denote alternatives and parameters weighting as actual numbers, satisfying Δj> 0,∇i> 0 and j=1nΔj=1,i=1m∇i=1. Suppose Fm×n= αij=ξij,Hij represents the cubic hesitant fuzzy soft decision matrix, where ξij and Hij denote the interval valued hesitant fuzzy soft set and Hij hesitant fuzzy soft set, correspondingly, which decision makers suggested. The following objectives are involved in using the CHFSWA (or CHFSWG) operator to solve MADM problems.

2.30. Algorithm

Step 1. Gain decision matrices for each alternate αij=ξij,Hij in form of CHFSNs for substitutes relative to attributes.

Step 2. Normalize the decision matrix to convert the assessment value of cost-type parameters into benefit-type strictures by using the normalization formula:

Mij=&αijc=1-ξij,1-Hij , for cost-type parameters&αij=ξij,Hij, for benefit-type parameters

Step3. Use the developed CHFSWA and CHFSWG operators to aggregate the CHFSNS αij=ξij,Hij for each alternative Ai={A1,A2,A3,....,An}.

CHFSWAα11,α12,...,αij=⊕j=1nΔj⊕i=1m∇i αij=1-j=1ni=1m1-ξijl∇iΔj,1-j=1ni=1m1-ξiju∇iΔj,j=1ni=1mHij∇iΔj

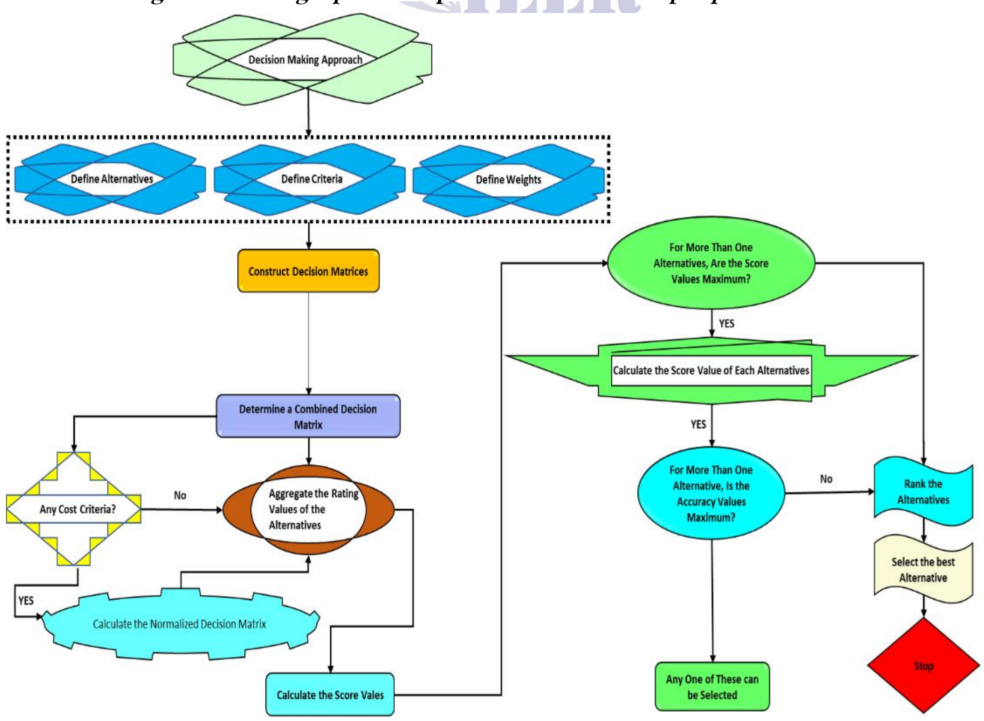
Alternatively, if we choose the CHFSWA operator:

CHFSWGα11,α12,...,αij=⊗j=1nΔj⊗i=1m∇i αij=j=1ni=1mξijl∇iΔj,j=1ni=1mξiju∇iΔj,1-j=1ni=1m1-Hij∇iΔj

Step 4. Calculate score values for each alternative using equation (8-9).

Step 5. Select most feasible alternative with the maximum score value.

Figure 2. The graphical representation of the proposed model



2.31. Case study

Scenario:

A cooperative of agriculturalists is determining which crop to plant on a large shared plot of land. They need to consider several criteria and gather input from different experts. The goal is to select the most suitable crop based on these criteria and expert opinions.

Criteria (Parameters):

i. Profit Potential (e_1) The estimated profit per acre from selling the crop.

ii. Water Requirements (e_2) The amount of water needed for the crop.

iii. Growth Duration (e_3) The time required for the crop to mature.

iv. Market Demand (e_4) The current market demand for the crop.

Alternatives:

i. A_1**:** Wheat

ii. A_2**:** Corn

iii. A_3**:** Soybeans

iv. A_4**:** Rice

Experts:

i. $\underset{\sim}{E}_1$**:** An agronomist

ii. $\underset{\sim}{E}_2$**:** An economist

iii. $\underset{\sim}{E}_3$**:** A hydrologist

iv. $\underset{\sim}{E}_4$**:** A market analyst

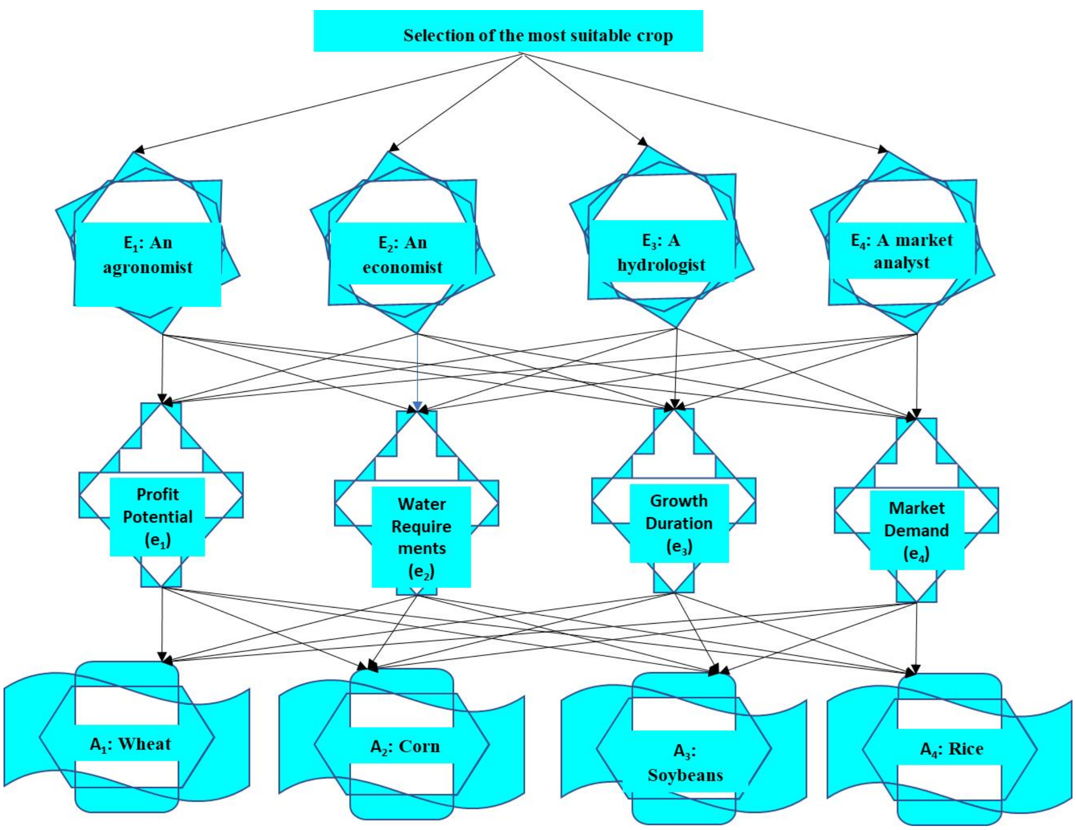


Table 1: Matrix of Decision For A1

Substitutes	**è1**	**è2**	**è3**	**è4**
$\underset{\sim}{E}_1$	0.1,0.2,0.1,0.2	0.2,0.5,0.2,0.3	0.2,0.7,0.1,0.3	0.2,0.5,0.3,0.7
$\underset{\sim}{E}_2$	0.3,0.4,0.1,0.7	0.1,0.9,0.6,0.8	0.1,0.3,0.4,0.5	0.3,0.4,0.2,0.3
$\underset{\sim}{E}_3$	0.1,0.3,0.5,0.6	0.2,0.3,0.1,0.7	0.2,0.3,0.1,0.3	0.2,0.3,0.1,0.3

$\underset{\sim}{E}_4$	0.2,0.3,0.7,0.8	0.2,0.3,0.1,0.4	0.1,0.8,0.2,0.8	0.2,0.5,0.1,0.6

Table 2: Matrix of Decision For A2

Substitutes	è1	è2	è3	è4
$\underset{\sim}{E}_1$	0.1,0.7,0.1,0.8	0.2,0.3,0.3,0.7	0.3,0.5,0.1,0.7	0.2,0.3,0.1,0.6
$\underset{\sim}{E}_2$	0.2,0.6,0.1,0.8	0.1,0.6,0.1,0.3	0.1,0.5,0.1,0.4	0.1,0.2,0.1,0.6
$\underset{\sim}{E}_3$	0.1,0.5,0.1,0.3	0.2,0.3,0.2,0.8	0.2,0.5,0.1,0.9	0.1,0.4,0.1,0.9
$\underset{\sim}{E}_4$	0.2,0.7,0.1,0.7	0.2,0.4,0.1,0.8	0.2,0.4,0.1,0.9	0.1,0.7,0.1,0.7

Table 3: Matrix of Decision For A3

Substitutes	è1	è2	è3	è4
$\underset{\sim}{E}_1$	0.3,0.4,0.4,0.3	0.2,0.8,0.1,0.9	0.3,0.5,0.1,0.9	0.2,0.8,0.3,0.8
$\underset{\sim}{E}_2$	0.1,0.2,0.1,0.3	0.1,0.2,0.1,0.2	0.2,0.8,0.1,0.8	0.3,0.8,0.3,0.7
$\underset{\sim}{E}_3$	0.1,0.2,0.1,0.3	0.1,0.2,0.1,0.2	0.1,0.2,0.1,0.3	0.1,0.2,0.1,0.2
$\underset{\sim}{E}_4$	0.1,0.2,0.1,0.3	0.1,0.2,0.2,0.3	0.1,0.2,0.1,0.3	0.1,0.2,0.1,0.3

Table 4: Matrix of Decision For A4

Substitutes	è1	è2	è3	è4
$\underset{\sim}{E}_1$	0.2,0.6,0.3,0.5	0.3,0.8,0.1,0.8	0.3,0.5,0.1,0.9	0.2,0.8,0.3,0.8
$\underset{\sim}{E}_2$	0.1,0.3,0.2,0.3	0.2,0.4,0.2,0.6	0.2,0.8,0.1,0.8	0.3,0.8,0.3,0.7
$\underset{\sim}{E}_3$	0.1,0.2,0.1,0.6	0.2,0.4,0.3,0.5	0.1,0.2,0.1,0.3	0.1,0.2,0.1,0.2
$\underset{\sim}{E}_4$	0.3,0.4,0.1,0.4	0.2,0.4,0.3,0.5	0.2,0.4,0.2,0.3	0.1,0.3,0.1,0.4

i. **Benefit Criteria:** Profit Potential (e_1), Market Demand (e_4)

ii. **Cost Criteria:** Water Requirements (e_2), Growth Duration (e_3)

Table 5: Decision Matrix of Normalized For A1

Substitutes	**è1**	**è2**	**è3**	**è4**
H^1	0.1,0.2,0.1,0.2	0.3,0.8,0.8,0.7	0.3,0.7,0.1,0.3	0.2,0.5,0.3,0.7
H^2	0.3,0.4,0.1,0.7	0.1,0.9,0.6,0.8	0.7,0.9,0.4,0.5	0.3,0.4,0.2,0.3
H^3	0.1,0.3,0.5,0.6	0.7,0.8,0.9,0.3	0.7,0.8,0.9,0.7	0.2,0.3,0.1,0.3
H^4	0.2,0.3,0.7,0.8	0.7,0.8,0.9,0.6	0.2,0.9,0.9,0.7	0.2,0.5,0.1,0.6

Table 6: Decision Matrix of Normalized For A2

Substitutes	è1	è2	è3	è4
H^1	0.1,0.7,0.1,0.8	0.7,0.8,0.3,0.7	0.5,0.7,0.1,0.7	0.2,0.3,0.1,0.6
H^2	0.2,0.6,0.1,0.8	0.4,0.6,0.1,0.3	0.5,0.9,0.1,0.4	0.1,0.2,0.1,0.6
H^3	0.1,0.5,0.1,0.3	0.7,0.8,0.2,0.8	0.5,0.7,0.1,0.9	0.1,0.4,0.1,0.9
H^4	0.2,0.7,0.1,0.7	0.6,0.8,0.1,0.8	0.6,0.8,0.1,0.9	0.1,0.7,0.1,0.7

Table 7: Decision Matrix of Normalized For A3

Substitutes	è1	è2	è3	è4
H^1	0.3,0.4,0.4,0.3	0.2,0.8,0.9,0.1	0.5,0.7,0.9,0.1	0.2,0.8,0.3,0.8
H^2	0.1,0.2,0.1,0.3	0.8,0.9,0.9,0.8	0.2,0.8,0.9,0.2	0.3,0.8,0.3,0.7
H^3	0.1,0.2,0.1,0.3	0.8,0.9,0.9,0.8	0.8,0.9,0.9,0.7	0.1,0.2,0.1,0.2
H^4	0.1,0.2,0.1,0.3	0.8,0.9,0.8,0.7	0.8,0.9,0.9,0.7	0.1,0.2,0.1,0.3

Table 8: Decision Matrix of Normalized For A4

Substitutes	è1	è2	è3	è4
H[1]	0.2,0.6,0.3,0.5	0.2,0.7,0.9,0.2	0.5,0.7,0.9,0.1	0.2,0.8,0.3,0.8
H[2]	0.1,0.3,0.2,0.3	0.6,0.8,0.8,0.4	0.2,0.8,0.9,0.2	0.3,0.8,0.3,0.7
H[3]	0.1,0.2,0.1,0.6	0.6,0.8,0.7,0.5	0.8,0.9,0.9,0.7	0.1,0.2,0.1,0.2
H[4]	0.3,0.4,0.1,0.4	0.6,0.8,0.7,0.5	0.6,0.8,0.8,0.7	0.1,0.3,0.1,0.4

4. Applying CHFSWA

On applying step 3 of the procedure cumulative the CHFSS, decision matrix denoted in Table 1 for each substitute. Like:

For membership interval valued hesitant fuzzy set:

CHFSWAα11,α12,…,αij= ⊕j=1nΔj ⊕i=1m∇i αij=1-j=1ni=1m(1-ξijl)∇iΔj,1-j=1ni=1m(1-ξiju)∇iΔj

For hesitant fuzzy set non-membership

CHFSWAα11,α12,…,αij= ⊕j=1nΔj ⊕i=1m∇i αij= j=1ni=1mHij∇iΔj

The whole accumulated table for CHFSWA Operator denoted in Tables 2–4.

Table 9: Result we get with *CHFSWA* coordinator against *A1*

Wight	e1	e2	e3	e4
0.25,0.21,0.25,,0.29T	0.4532	0.5653	0.5323	0.6625
0.32,0.20,0.28,0.20	0.5532	0.5518	0.6671	0.4529
0.25,0.5,0.42,0.18	0.6672	0.6431	0.9721	0.5383
0.20,0.10,0.22,0.38	0.7789	0.6432	0.5519	0.7214

Table 10: Result we get with *DHFSYWA* coordinator against *A2*

	e1	e2	e3	e4
0.29,0.25,0.21,0.25T	0.6421	0.5537	0.7756	0.87891
0.28,0.20,0.20,0.32	0.5409	0.5460	0.6537	0.6432
0.18,0.25,0.42,0.5	0.6634	0.6349	0.5476	0.7719
0.38,0.20,0.10,0.22	0.6648	0.5576	0.6892	0.6629

Table 11: Result we get with *DHFSYWA* coordinator against *A3*

	e1	e2	e3	e4
0.29,0.25,0.21,0.25T	0.5525	0.4344	0.7629	0.6689
0.28,0.20,0.32,0.20	0.5328	0.6627	0.6566	0.6643
0.5,0.42,0.18,0.25	0.6521	0.4479	0.5418	0.5519
0.38,0.20,0.22,0.10	0.5344	0.5531	0.5467	0.4429

Table 12: Result we get with *DHFSYWA* coordinator against *A4*

	e1	e2	e3	e4
0.29,0.25,0.21,0.25T	0.5527	0.5734	0.5563	0.6643
0.32,0.20,0.28,0.20	0.4478	0.6537	0.4409	0.5531
0.25,0.5,0.42,0.18	0.5418	0.7723	0.7710	0.5222
0.38,0.20,0.22,0.10	0.6667	0.5228	0.6698	0.5433

Now, by applying step 4 of the procedure calculate the grade assessment, denoted in Table 5 and mark the whole replacements correspondingly.

Table 13:

	Grade Assessment
P1	0.5678
P2	0.7891
P3	0.6719
P4	0.8834

Ranking order P4⩾ P2⩾ P3⩾ P1

5. Applying CHFSOWA

On applying step 3 of the procedure cumulative the CHFS, conclusion matrix denoted in Table 1 for each substitute. Like:

For membership interval hesitant fuzzy set:

CHFSWAα11,α12,…,αij=⊕j=1nΔj⊕i=1m∇i αij=1-j=1ni=1m1-ξijl∇iΔj,1-j=1ni=1m1-ξiju∇iΔj

For non-membership

CHFSWAα11,α12,…,αij=⊕j=1nΔj⊕i=1m∇i αij=j=1ni=1mHij∇iΔj

The whole accumulated table for DHFSYOWA Operator denoted in Tables 2–4.

Table 14: Result we get with *DHFSYOWA* coordinator against *A1*

Wight	e1	e2	e3	e4
0.25,0.21,0.25,,0.29T	0.3427	0.6519	0.5523	0.7625
0.32,0.20,0.28,0.20	0.4532	0.4518	0.5671	0.5329
0.25,0.5,0.42,0.18	0.5672	0.4431	0.8723	0.4389
0.20,0.10,0.22,0.38	0.4789	0.5432	0.4519	0.6234

Table 15: Result we get with *DHFSYOWA* coordinator against *A2*

Wight	**e1**	**e2**	**e3**	**e4**
0.25,0.21,0.25,,0.29T	0.5421	0.5437	0.8756	0.6891
0.32,0.20,0.28,0.20	0.4509	0.4560	0.4537	0.5432
0.25,0.5,0.42,0.18	0.7634	0.6549	0.3476	0.6719
0.20,0.10,0.22,0.38	0.5248	0.4576	0.7892	0.5629

Table 16: Result we get with *DHFSYOWA* coordinator against *A3*

Wight	**e1**	**e2**	**e3**	**e4**
0.25,0.21,0.25,,0.29T	0.4455	0.3344	0.6619	0.7589
0.32,0.20,0.28,0.20	0.4328	0.4627	0.5466	0.5643
0.25,0.5,0.42,0.18	0.5521	0.3479	0.5618	0.4519
0.20,0.10,0.22,0.38	0.3344	0.4531	0.4567	0.4329

Table 17: Result we get with *DHFSYOWA* coordinator against *A4*

Wight	e1	e2	e3	e4
0.25,0.21,0.25,,0.29T	0.4327	0.6734	0.4563	0.6643
0.32,0.20,0.28,0.20	0.3478	0.5437	0.4509	0.4431
0.25,0.5,0.42,0.18	0.4318	0.7623	0.7610	0.5522
0.20,0.10,0.22,0.38	0.4567	0.5428	0.5498	0.4433

Now, by applying step 4 of the procedure calculate the grade assessment, represented in **Table 18:**

Table 5 and mark the whole replacements correspondingly.

	Grade Assessment
A1	0.6654
A2	0.8891
A3	0.7628
A4	0.9134

Ranking order A4⩾ A2⩾ A3⩾ A1

6. Applying CHFSWG

On applying step 3 of the procedure cumulative the DHFS decision matrix denoted in Table 1 for each substitute. Like:

For membership hesitant fuzzy set:

DHFSYHWA(P1)=j=1mψj i=1n(ϕiηΦ(ij))=(HΦij,℘ Φ(ij))∈ηΦ(ij)min1,j=1m ψji=1nϕi.HΦ(ij)λ1λ,

DHFSYHWA(P1)=j=1mψj i=1n(ϕiP1)=(Hij, ℘ ij)∈ηijmin1,j=1mψji=1nϕi.Hijλ1λ=0.4312674 3

For non-membership

DHFSYHWA P1=j=1mψj i=1nϕiP1=Hij,℘ ij∈ηij1-min1,j=1mψji=1nϕi.1-℘ ijλ1λ=0.567194.

The whole accumulated table for DHFSYHWA, Coordinator represented in Tables 2–4.

Table 19: Result we get with *DHFSYHWA* coordinator against *A1*

Wight	e1	e2	e3	e4
0.25,0.21,0.25,,0.29T	0.4532	0.5561	0.5531	0.8761
0.32,0.20,0.28,0.20	0.5567	0.5613	0.7612	0.6633
0.25,0.5,0.42,0.18	0.4455	0.5577	0.7766	0.4519
0.20,0.10,0.22,0.38	0.5421	0.6663	0.6633	0.7613

Table 20: Result we get with *DHFSYHWA* coordinator against *A2*

Wight	**e1**	**e2**	**e3**	**e4**
0.25,0.21,0.25,,0.29T	0.5566	0.6613	0.7765	0.7789
0.32,0.20,0.28,0.20	0.4433	0.4432	0.7745	0.5576
0.25,0.5,0.42,0.18	0.6712	0.6654	0.5543	0.6621
0.20,0.10,0.22,0.38	0.4512	0.5544	0.7766	0.3321

Table 21: Result we get with *DHFSYHWA* coordinator against *A3*

Wight	**e1**	**e2**	**e3**	**e4**
0.25,0.21,0.25,,0.29T	0.4455	0.3344	0.7513	0.6513
0.32,0.20,0.28,0.20	0.6754	0.5613	0.7241	0.8723
0.25,0.5,0.42,0.18	0.6864	0.6752	0.8614	0.5613
0.20,0.10,0.22,0.38	0.6518	0.5671	0.5713	0.7513

Table 22: Result we get with *DHFSYHWA* coordinator against *A4*

Wight	**e1**	**e2**	**e3**	**e4**
0.25,0.21,0.25,,0.29T	0.4327	0.6544	0.7311	0.6323
0.32,0.20,0.28,0.20	0.3478	0.5327	0.5439	0.4321

0.25,0.5,0.42,0.18	0.5418	0.7433	0.3340	0.5122
0.20,0.10,0.22,0.38	0.6567	0.5438	0.3428	0.4563

Now, by applying step 4 of the procedure calculate the grade assessment, denoted in Table 5 and mark the whole replacements correspondingly.

Table 23:

	Grade Assessment
A1	0.6018
A2	0.7651
A3	0.6891
A4	0.8761

Ranking order A4⩾ A2⩾ A3⩾ A1

7. Applying CHFSOWG

On applying step 3 of the procedure cumulative the DHFS decision matrix denoted in Table 1 for each substitute. Like:

For membership hesitant fuzzy set:

DHFSYHWA(P1)=j=1mψj i=1n(ϕiηΦ(ij))=(HΦij,℘ Φ(ij))∈ηΦ(ij)min1,j=1m ψji=1nϕi.HΦ(ij)λ1λ,

DHFSYHWA(P1)=j=1mψj i=1n(ϕiP1)=(Hij,℘ ij)∈ηijmin1,j=1mψji=1nϕi.Hijλ1λ=0.4312674 3

For non-membership

DHFSYHWA P1=j=1mψj i=1nϕiP1=Hij,℘ ij∈ηij1-min1,j=1mψji=1nϕi.1-℘ ijλ1λ=0.567194.

The total aggregated table for DHFSYHWA Operator denoted in Tables 2–4.

Table 19: Result we get with *DHFSYHWA* coordinator against *A1*

Wight	**e1**	**e2**	**e3**	**e4**
0.25,0.21,0.25,,0.29T	0.4532	0.5561	0.5531	0.8761
0.32,0.20,0.28,0.20	0.5567	0.5613	0.7612	0.6633
0.25,0.5,0.42,0.18	0.4455	0.5577	0.7766	0.4519
0.20,0.10,0.22,0.38	0.5421	0.6663	0.6633	0.7613

Table 20: Result we get with *DHFSYHWA* coordinator against *A2*

Wight	e1	e2	e3	e4
0.25,0.21,0.25,,0.29T	0.5566	0.6613	0.7765	0.7789
0.32,0.20,0.28,0.20	0.4433	0.4432	0.7745	0.5576
0.25,0.5,0.42,0.18	0.6712	0.6654	0.5543	0.6621
0.20,0.10,0.22,0.38	0.4512	0.5544	0.7766	0.3321

Table 21: Result we get with *DHFSYHWA* coordinator against *A3*

Wight	**e1**	**e2**	**e3**	**e4**
0.25,0.21,0.25,,0.29T	0.4455	0.3344	0.7513	0.6513
0.32,0.20,0.28,0.20	0.6754	0.5613	0.7241	0.8723
0.25,0.5,0.42,0.18	0.6864	0.6752	0.8614	0.5613
0.20,0.10,0.22,0.38	0.6518	0.5671	0.5713	0.7513

Table 22: Result we get with *DHFSYHWA* coordinator against *A4*

Wight	e1	e2	e3	e4
0.25,0.21,0.25,,0.29T	0.4327	0.6544	0.7311	0.6323
0.32,0.20,0.28,0.20	0.3478	0.5327	0.5439	0.4321
0.25,0.5,0.42,0.18	0.5418	0.7433	0.3340	0.5122
0.20,0.10,0.22,0.38	0.6567	0.5438	0.3428	0.4563

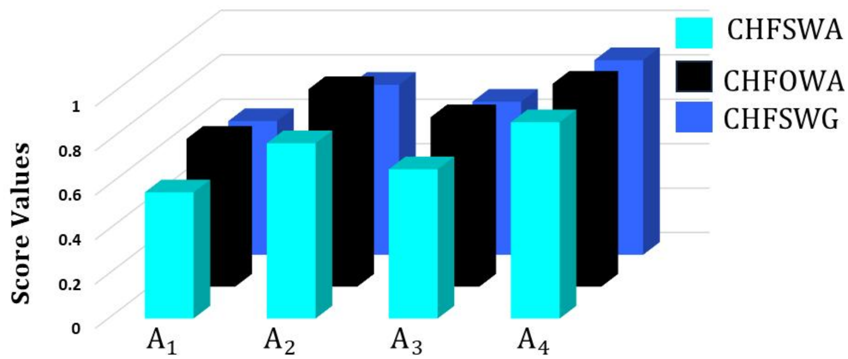
Now, by applying step 4 of the procedure calculate the grade assessment, denoted in Table 5 and mark the whole replacements correspondingly.

Table 23:

	Grade Assessment
A1	0.6018
A2	0.7651
A3	0.6891
A4	0.8761

Ranking order A4⩾ A2⩾ A3⩾ A1

Figure 2



A. **TEST OF VALIDITY:**

In establishing the suppleness and healthiness about projected skill, we follow the assessment protocols proven by Wang and Triantaphyllou [32] through a series of difficult steps:

Step 1: The leading validation contains replacing the ratings of suboptimal substitutes with those of lower-quality ones while ensuring that the highest-rated alternative remains same. This step checks if the model regularly recognizes the top alternative, thus representing its stability in decision-making, even when differences occur among subordinates substitute.

Step 2: The next confirmation follows norm of transitivity. This norm dictates that if substitute A is favored over substitute Ḃ, and Ḃ is preferred over substitute Ċ, then Â should logically be preferred over Ċ. Guaranteeing this rational reliability in the ranking process endorses that the model maintains a coherent preference order across diverse decision situations.

Step 3: The additional confirmation checks the model's reliability when the judgment difficulty is distributed into smaller sub-problems. It confirms that the early ranking of alternatives is unspoiled within these sub-problems, thereby approving the model's consistency and vigor in intricate, fragmented decision-making situations.

By following these assessment protocols, the suggested technique shows its legitimacy and steadfastness across several decision-making circumstances:

- It regularly recognizes the highest substitute even when non-optimal alternatives are changed.
- It follows to the norm of transitivity, certifying a coherent and logical inclination order.
- It maintains the early level of substitutes when decision difficult is partitioned into minor fragments.

Test of Validity Using Criterion 1: The preliminary ranking of alternatives is A4⩾ A2⩾ A3⩾ A1 . To endorse this, we replaced the non-optimal substitute A1 by a substandard substitute A1* and recalculated rankings. The aggregated scores for A4⩾ A2⩾ A3⩾ A1* resulted in a reread ranking of A4⩾ A2⩾ A3⩾ A1* , confirming the method's sturdiness in identifying the best substitute notwithstanding substitutions.

Test of Validity Using Criteria 2 and 3: Added assessment involved disjointed decision-making developments: {A4,A2,A3},{A1,A2,A3}, and {A1,A2,A4}. These subcases were ranked steadily with the initial process, reaffirming logical uniformity (transitivity) and permanency in decision rankings.

Merging all the findings, the overall ranking remain, A4⩾ A2⩾ A3⩾ A1 , which aligns exactly with the results of the primary policymaking process. This reliability confirms that our planned strategy meets the requirements for maintaining logical consistency (transitivity) and stability when the problem is divided into minor parts.

B. MANAGERIAL IMPLICATIONS:

Introduction of dual hesitant fuzzy Yager accumulation facilitator into multi-ascribe policymaking processes has profound managerial implications across diverse industries. These operators, grounded in Yager's directives families of t-standards and t-costandards, provide enhanced flexibility and precision in evaluating complex decision scenarios involving multiple attributes with varying degrees of hesitation and fuzziness.

Managers can leverage these aggregation tools to make more informed and strategic decisions in environments characterized by uncertainty and varying levels of importance across criteria. For instance, in strategic planning and resource allocation, these operators allow decision-makers to effectively weigh and balance diverse factors such as cost, risk, and potential returns. The flexibility offered by the Yager operators is particularly beneficial in scenarios where different criteria or stakeholder preferences need to be prioritized dynamically according to changing business conditions.

By applying these advanced aggregation methods, organizations can improve their decision-making processes, leading to exact and authentic outcomes. This capability is serious for maintaining competitiveness in fast-paced markets where decisions must be made swiftly and based on complex, often conflicting, information. In addition, the robustness of these operators ensures that decisions remain consistent and logical in spite of the decision difficulty is fragmented declining to smaller, other controllable sub-problems.

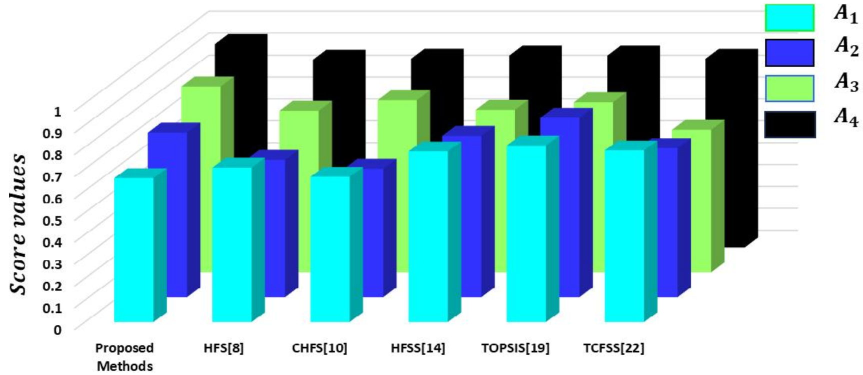
In practical applications, such as in project management, supply chain optimization, or product development, the use of these Yager-based aggregation operators enables managers to optimize their strategies, align decisions with long-term objectives, and enhance overall organizational performance. The ability to apply these methods effectively can lead to better resource utilization, improved operational efficiency, and a stronger alignment of decisions with organizational goals, thereby driving success in competitive and dynamic business environments.

1.1 Comparison

Table 11 Represent the Comparison of Different methods based on score values

Methods	A1	A2	A3	A4	Ranking Order
Proposed Methods	0.4582	0.5121	0.7503	0.8293	A4⩾ A3⩾ A2⩾ A1
HFS[8]	0.6063	0.5279	0.6391	0.7891	A4⩾ A3⩾ A1⩾ A2
CHFS[10]	0.5643	0.5078	0.7867	0.8234	A4⩾ A3⩾ A1⩾ A2
HFSS[14]	0.7803	0.6376	0.7433	0.8179	A4⩾ A1⩾ A3⩾ A2
TOPSIS[19]	0.6975	0.7204	0.5775	0.8778	A4⩾ A2⩾ A1⩾ A3
TCFSS[22]	0.7349	0.6418	0.5512	0.8632	A4⩾ A1⩾ A2⩾ A3

Figure 5: The graphical representation of different approaches



3.1. Sensitivity Analysis

In current review, this developed ideal is thoroughly verified through couple of various understanding studies, either aiming upon variations in standards and policymaking loads and their consequent outcome on the last rankings contained by a cubic hesitant fuzzy soft domain. This initial understanding study consist of the sequential assessment in order to inspect in what way diverse relative levels allotted to mention benchmarks—top, alike, and truncated—mark that entire ranking results. The said inquiry involves running that ideal independently on behalf of apiece criterion, assigning locus loads independently inside that cubic hesitant fuzzy soft structure. Outcomes, indicated in the Figure 4, originate from 20 poles apart scenarios. Oddly, in every illustration, unconventional A4consistently arises as the top-ranked option, while alternative A1 invariably ranks at the end. In spite of significant deviations for benchmark bulks, this prototypical reveals an important tactlessness to those modifications, stressing his robustness in cubic hesitant fuzzy soft circumstances.

Another enquiry focuses upon correcting the loads given to the subjects in the cubic hesitant fuzzy soft context, ensuing in 4 diverse situations with varied weightiness divisions. Domino effect of those situations displayed in Fig (5). Unswervingly, across total situations, unusual

A4 is reaffirmed as maximum favored choice, whereas unconventional A1 leftovers as minimum favored. Although, relative standings of further substitutes might change reliant on the maker masses employed, the scheduled prototypical verifies a robust as well as trustworthy across a wide array of weighting formations in a cubic hesitant fuzzy soft arrangement.

These studies prove the resilience and reliability of intended ideal in managing variations in mutual criteria encumbrances and decision choices. This integration of cubic hesitant fuzzy soft sets shows propositions of a more complex and supple method for conclusion, permitting in lieu of consideration of equally affirmative and deleterious membership grades. These memberships ability improves the facsimile's capability to seize and method intricate, practical deciding situations, hence giving further accurate and inclusive valuations. Additionally, the prototype's callousness to chief deviations in standard weightiness and makers inclinations underscores his toughness, which is vital in term of applied uses where executive environment is a lot subject to active plus random fluctuations. The cubic hesitant fuzzy soft set context guarantees ideal's durability, despite changing circumstances.

Wide-ranging scrutiny too displays the models prospective in lieu of wider suitability across diverse range. Authentication of the model's enactment through several distribution of weight, analysis also assures its proficiency as well as flexibility, creating more appropriate for a varied applicability, ranging from medicinal features to variety of decision-making in diverse field. Upcoming research should concentrate on further intensifying the idea's aptness and refining their parameters to warrant optimal presentation through a more extensive realm of policymaking situations.

Table 9: Exhibit different sets of masses designate to conditions.

W	ƛ1	ƛ2	ƛ3	ƛ4	W	ƛ1	ƛ2	ƛ3	ƛ4	W	ƛ1	ƛ2	ƛ3	ƛ4
$\dot{T}_{1}$	0.2	0.4	0.3	0.1	$\dot{T}_{9}$	0.1	0.2	0.1	0.6	$\dot{T}_{17}$	0.1	0.2	0.4	0.3
$\dot{T}_{2}$	0.4	0.1	0.2	0.3	$\dot{T}_{10}$	0.2	0.1	0.4	0.3	$\dot{T}_{18}$	0.4	0.2	0.2	0.2
$\dot{T}_{3}$	0.2	0.1	0.3	0.4	$\dot{T}_{11}$	0.1	0.2	0.3	0.4	$\dot{T}_{19}$	0.3	0.3	0.3	0.1
$\dot{T}_{4}$	0.2	0.3	0.1	0.4	$\dot{T}_{12}$	0.2	0.2	0.3	0.3	$\dot{T}_{20}$	0.3	0.2	0.2	0.3
$\dot{T}_{5}$	0.4	0.1	0.1	0.4	$\dot{T}_{13}$	0.2	0.2	0.2	0.4	$\dot{T}_{21}$	0.1	0.2	0.3	0.4
$\dot{T}_{6}$	0.2	0.3	0.2	0.3	$\dot{T}_{14}$	0.4	0.1	0.3	0.2	$\dot{T}_{22}$	0.1	0.2	0.3	0.4
$\dot{T}_{7}$	0.1	0.2	0.2	0.5	$\dot{T}_{15}$	0.1	0.2	0.4	0.3	$\dot{T}_{23}$	0.1	0.4	0.2	0.3
$\dot{T}_{8}$	0.2	0.1	0.1	0.6	$\dot{T}_{16}$	0.2	0.3	0.4	0.1	$\dot{T}_{24}$	0.2	0.2	0.2	0.4

Spectrum of Engineering Sciences
ISSN (e) 3007-3138 (p) 3007-312X Volume 4, Issue 4, 2026

Figure 4: Displays the various sets of values of alternatives weights assigned to criteria

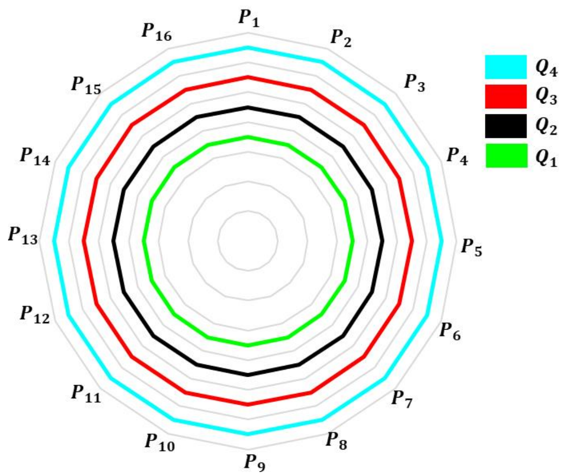
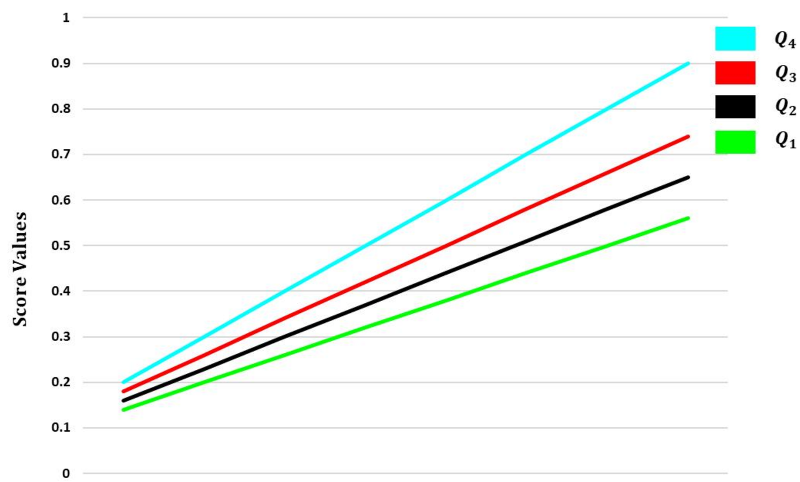


Table 10: presents various sets of masses assigned by policy-makers

W	M1	M2	M3	M4	W	ϑ1	ϑ2	ϑ3	M4	W	ϑ1	ϑ2	ϑ3	M4
$\dot{T}_1$	0.1	0.2	0.3	0.4	$\dot{T}_9$	0.2	0.2	0.2	0.4	$\dot{T}_{17}$	0.1	0.2	0.3	0.4
$\dot{T}_2$	0.3	0.2	0.4	0.1	$\dot{T}_{10}$	0.3	0.3	0.3	0.1	$\dot{T}_{18}$	0.3	0.2	0.3	0.2
$\dot{T}_3$	0.2	0.3	0.3	0.2	$\dot{T}_{11}$	0.4	0.2	0.3	0.1	$\dot{T}_{19}$	0.2	0.2	0.1	0.5
$\dot{T}_4$	0.1	0.2	0.4	0.3	$\dot{T}_{12}$	0.2	0.1	0.6	0.1	$\dot{T}_{20}$	0.3	0.2	0.1	0.4
$\dot{T}_5$	0.1	0.2	0.4	0.3	$\dot{T}_{13}$	0.3	0.2	0.4	0.1	$\dot{T}_{21}$	0.1	0.3	0.3	0.3
$\dot{T}_6$	0.3	0.4	0.2	0.1	$\dot{T}_{14}$	0.3	0.4	0.2	0.1	$\dot{T}_{22}$	0.1	0.2	0.2	0.5
$\dot{T}_7$	0.4	0.4	0.1	0.1	$\dot{T}_{15}$	0.2	0.5	0.2	0.1	$\dot{T}_{23}$	0.2	0.2	0.4	0.2
$\dot{T}_8$	0.4	0.3	0.2	0.1	$\dot{T}_{16}$	0.2	0.4	0.2	0.2	$\dot{T}_{24}$	0.4	0.3	0.1	0.2

Figure 3: Alternative classification with varying criteria weights



Discussion

The overview of cubic hesitant fuzzy soft sets (CHFSS) denotes an essential progress within area of fuzzy set ideas, specifically in a perspective of multi-criteria decision-making (MCDM). In joining a significant of cubic sets and hesitant fuzzy soft sets, CHFSS offers an additional nuanced approach to handling ambiguity, one that accommodates both the hesitancy essential in decision-making processes and the multidimensional nature of the standards involved. This hybrid structure allows for a comfortable demonstration of information, which is central in situations where traditional fuzzy sets may descend short. The academic improvement of this study are chiefly remarkable, as they not only represent an advance mathematical structure but also spread the effectiveness of CHFSS through the growth of particular aggregation operators. The Cubic HFSWA, Cubic HFSWG, Cubic HFSOWA, and Cubic HFSOWG facilitator contributes forceful instrument for collecting data in situations characterized by high stages of ambiguity and inaccuracy. The hard inquiry of these operators' features enhances a layer of consistency to their application, ensuring that decision-makers can trust the results created by these procedures. Furthermore, the overview of hybrid operators more enriches the CHFSS structure, enabling challenging and custom-made aggregation policies that can well align with the precise needs of numerous decision-making situations. This flexibility is mainly important in real-world applications, where decision-making frequently involves balancing several, sometimes contradictory, criteria. The systematic analysis of these operators' properties not only establishes their theoretical accuracy but also offers practical insights into their application.

To summarize, the CHFSS structure and its related aggregation operators offer a commanding new approach to dealing with uncertainty in MCDM scenarios. This study give a strong theoretical foundation for upcoming research, suggesting several possibilities for more detail, such as the application of CHFSS in diverse domains and the development of additional operators to demonstrates specific decision-making challenges. The advanced contributions made in this study have the prospective to significantly improve decision-making processes in a wide range of fields, from economics and engineering to environmental management and beyond.

6.1 Limitation

Regardless of the importance progress brought in by the cubic hesitant fuzzy soft set (CHFSS) model, various restrictions guarantee consideration. Initially, the intricacy of the mathematical structure and the novel improved aggregation operators may present hurdles over computational efficiency. The complex behavior of CHFSS, specifically when acting with huge datasets or a high number of criteria, could guide to raise computational consequences and duration, which may restrict its applicability in time-sensitive decision-making situations.

Another, practicality of CHFSS in real-world situations is yet in its initial stages of discovery. Although the theoretical base is rigorous, empirical validation around different structures is important to wholly interpret its specialty and restrictions in pragmatically applications. The execution of CHFSS-based methods with respect to present fuzzy set idea under diverse states stay an area that needs more study.

Furthermore, the structure assumes a definite degree of skillfulness in fuzzy set theory and corresponding mathematical ideas, which may restrict its approachability to practician who are not mastered in these. The complication of the operators and the demand of the profound interpretation of these applications make high learning curve for users, possibly preventing its wider selection.

Lastly, while the CHFSS structure suggests an additional nuanced method to managing improbability and fuzziness, it may still confront problems when treating with highly imprecise or extremely dynamic situations where the sort of unpredictability is regularly changing. The present operators, though flexible, may need more modification or the advancement of extra tools to wholly seize the complication of such situations.

In short, while the CHFSS structure give a favorable development in decision-making under doubt, its restrictions, specifically in terms of computational complication, pragmatic authentication, approachability, and flexibility to highly changing situations, must be managed in upcoming study to capitalize on its applied efficacy and impression.

6.2 Advantages

The outline of cubic hesitant fuzzy soft sets (CHFSS) proposes numerous important significances that improve its applicability and efficiency in difficult decision-making scenarios:

i. **Enhanced Flexibility and Representation**: The CHFSS structure chains the values of cubic sets and hesitant fuzzy soft sets, giving an extra wide-ranging depiction of ambiguity and fuzziness. This dual-level suppleness lets decision-makers to model difficult situations extra precisely, capturing both the hesitancy in skilled views and the multidimensional nature of criteria involved.

ii. **Robust Aggregation Operators**: The improvement of expert aggregation operators within the CHFSS structure, such as the Cubic HFSWA, Cubic HFSWG, Cubic HFSOWA, and Cubic HFSOWG, suggest dominant tools for creating statistics. These operators are intended to cope with the complexities of hesitant and cubic fuzzy statistics, leading to more trustworthy and nuanced decision results.

iii. **Applicability in Multi-Criteria Decision-Making (MCDM)**: CHFSS is specifically beneficial in MCDM situations where numerous standards need to be assessed instantaneously. The context's capability to face intricate, ambiguous, and inexact statistics makes it ideal for situations where customary fuzzy set concepts may be short.

iv. **Customizable Hybrid Operators**: The overview of hybrid operators within the CHFSS structure enhances an additional level of customization, letting decision-makers to shape the aggregation procedure according to particular requirements. This malleability improves the structure's flexibility, making it appropriate for a comprehensive range of decision-making perspectives, from environmental controlling to economics.

v. **Theoretical Rigor and Reliability**: The difficult study of the characteristics of CHFSS aggregation operators guarantees that the decision-making procedures grounded on this context are theoretically comprehensive. This reliability is crucial for operations in fields where purposes have significant pretenses, such as healthcare, engineering, and finance.

vi. **Improved Handling of Uncertainty**: CHFSS delivers a more intricate manner to dealing with uncertainty by integrating both the hesitant and cubic appearance of fuzzy items. This purport for a more precise study of real-world stipulation, where dubiety is often complex and not readily attained by simpler models.

vii. **Foundation for Further Research**: The outcomes of the CHFSS configuration lay a cogent foundation for posterity research, propose multifold occasions to examine its applications about various spheres, refine its operators, and expand its theoretical underpinnings. This potential for continued development and innovation is a significant advantage for researchers and practitioners alike.

CONCLUSIONS

In this study, a novel concept of Cubic Hesitant Fuzzy Sets (CHFSs) has been proposed by effectively integrating the principles of cubic sets and hesitant fuzzy sets. This integration enables the formulation and investigation of fundamental operational laws within a unified mathematical framework, thereby enhancing the conceptual coherence of fuzzy set theory. Several new aggregation operators have been developed, including cubic hesitant fuzzy soft weighted averaging and geometric operators, ordered weighted averaging and geometric operators, as well as hybrid averaging and geometric operators. The mathematical properties of these operators have been thoroughly examined, demonstrating their capability to handle complex and uncertain decision-making problems.

The proposed framework extends and generalizes existing models such as fuzzy sets, intuitionistic fuzzy sets, hesitant fuzzy sets, and cubic fuzzy sets, providing a more robust and powerful tool for managing uncertainty and imprecision. These operators were employed within a multi-attribute decision-making environment, illustrated through a practical example involving the selection of the most suitable crop. The results highlight the effectiveness and applicability of the proposed approach. Furthermore, sensitivity analysis confirmed the stability and reliability of the obtained outcomes, while comparative analysis validated the superiority of the proposed method over existing decision-making techniques.

Future research directions include the application of cubic hesitant fuzzy soft sets to pattern recognition and medical diagnosis problems. Additionally, the development of distance and similarity measures for these sets will be explored to further enhance their applicability and practical relevance.

References

[1] L. A. Zadeh, "Fuzzy sets," *Information and control,* vol. 8, no. 3, pp. 338-353, 1965.

[2] M. B. Gorzałczany, "A method of inference in approximate reasoning based on interval-valued fuzzy sets," *Fuzzy sets and systems,* vol. 21, no. 1, pp. 1-17, 1987.

[3] Y. Jun, C. S. Kim, and K. O. Yang, "Annals of fuzzy mathematics and informatics," *Cubic Sets,* vol. 4, no. 1, pp. 83-98, 2011.

[4] V. Torra, "Hesitant fuzzy sets," *International journal of intelligent systems,* vol. 25, no. 6, pp. 529-539, 2010.

[5] M. Xia and Z. Xu, "Hesitant fuzzy information aggregation in decision making," *International journal of approximate reasoning,* vol. 52, no. 3, pp. 395-407, 2011.

[6] H. Liao and Z. Xu, "Subtraction and division operations over hesitant fuzzy sets," *Journal of intelligent & fuzzy systems,* vol. 27, no. 1, pp. 65-72, 2014.

[7] H. Liao and Z. Xu, "Some new hybrid weighted aggregation operators under hesitant fuzzy multi-criteria decision making environment," *Journal of intelligent & fuzzy systems,* vol. 26, no. 4, pp. 1601-1617, 2014.

[8] H. Liao and Z. Xu, "Extended hesitant fuzzy hybrid weighted aggregation operators and their application in decision making," *Soft computing,* vol. 19, pp. 2551-2564, 2015.

[9] G. Wei, X. Zhao, and R. Lin, "Some hesitant interval-valued fuzzy aggregation operators and their applications to multiple attribute decision making," *Knowledge-Based Systems,* vol. 46, pp. 43-53, 2013.

[10] T. Mahmood, F. Mehmood, and Q. Khan, "Cubic hesitant fuzzy sets and their applications to multi criteria decision making," *International Journal of Algebra and Statistics,* vol. 5, no. 1, pp. 19-51, 2016.

[11] T. Mahmood, F. Mehmood, and Q. Khan, "Some generalized aggregation operators for cubic hesitant fuzzy sets and their applications to multi criteria decision making," *Punjab University Journal of Mathematics*, vol. 49, no. 1, 2020.

[12] D. Molodtsov, "Soft set theory—first results," *Computers & mathematics with applications*, vol. 37, no. 4-5, pp. 19-31, 1999.

[13] P. K. Maji, R. Biswas, and A. Roy, "Fuzzy soft sets," 2001.

[14] K. Babitha and S. J. John, "Hesitant fuzzy soft sets," *Journal of New Results in Science*, vol. 2, no. 3, 2013.

[15] G. Muhiuddin and A. M. Al-roqi, "Cubic soft sets with applications in BCK/BCI-algebras," *Annals of Fuzzy Mathematics and Informatics*, vol. 8, no. 2, pp. 291-304, 2014.

[16] X. Yang, T. Y. Lin, J. Yang, Y. Li, and D. Yu, "Combination of interval-valued fuzzy set and soft set," *Computers & Mathematics with Applications*, vol. 58, no. 3, pp. 521-527, 2009.

[17] N. Chen, Z. Xu, and M. Xia, "Interval-valued hesitant preference relations and their applications to group decision making," *Knowledge-based systems*, vol. 37, pp. 528-540, 2013.

[18] S. Das and S. Kar, "The hesitant fuzzy soft set and its application in decision-making," in *Facets of Uncertainties and Applications: ICFUA, Kolkata, India, December 2013*, 2015: Springer, pp. 235-247.

[19] M. Akram and A. Adeel, "TOPSIS approach for MAGDM based on interval-valued hesitant fuzzy N-soft environment," *International Journal of Fuzzy Systems*, vol. 21, pp. 993-1009,

[20] M. J. Borah and B. Hazarika, "Some operators on interval-valued hesitant fuzzy soft sets," *Afrika Matematika*, vol. 29, pp. 509-529, 2018.

[21] W. Liang and Y.-M. Wang, "A probabilistic interval-valued hesitant fuzzy gained and lost dominance score method based on regret theory," *Computers & Industrial Engineering*, vol. 159, p. 107532, 2021.

[22] S. S. Appadoo, M. Makhan, and A. Kumar, "A note on "Dealer using a new trapezoidal cubic hesitant fuzzy TOPSIS method and application to group decision-making program"," *Soft Computing*, vol. 26, pp. 7139 - 7141, 2022.

[23] A. u. Rehman, M. Gulistan, N. Kausar, S. Kousar, M. M. A. Al-Shamiri, and R. Ismail, "Novel Development to the Theory of Dombi Exponential Aggregation Operators in Neutrosophic Cubic Hesitant Fuzzy Sets: Applications to Solid Waste Disposal Site Selection," *Complex.*, vol. 2022, pp. 3828370:1-3828370:16, 2022.

[24] D. Zhao, "Epidemiological features of cardiovascular disease in Asia," *JACC: Asia*, vol. 1, no. 1, pp. 1-13, 2021.

[25] D. Zhao, J. Liu, M. Wang, X. Zhang, and M. Zhou, "Epidemiology of cardiovascular disease in China: current features and implications," *Nature Reviews Cardiology*, vol. 16, no. 4, pp. 203-212, 2019.

[26] S. Rehman, E. Rehman, M. Ikram, and Z. Jianglin, "Cardiovascular disease (CVD): assessment, prediction and policy implications," *BMC Public Health*, vol. 21, pp. 1-14, 2021.

[27] M. Akram, U. Amjad, and B. Davvaz, "Decision-making analysis based on bipolar fuzzy N-soft information," *Computational and Applied Mathematics*, vol. 40, no. 6, p. 182, 2021.

[28] M. Akram, G. Ali, and M. Shabir, "A hybrid decision-making framework using rough m F bipolar soft environment," *Granular Computing*, vol. 6, pp. 539-555, 2021.