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DESIGN AND ANALYSIS OF HALBACH ARRAY PERMANENT MAGNET VERNIER MACHINE

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Abstract

In recent years, extensive research has been conducted on permanent magnet field machinery due to its ability to achieve high torque density. Increasing the average torque of electrical machines offers numerous advantages, including enhanced power output, improved performance, greater efficiency, compact structural design possibilities, and extended operational ranges. These advancements have significant implications for electric vehicles, renewable energy systems, industrial automation, and other emerging technologies, thereby driving overall technological progress. In this study, a ***Halbach array*** *has been introduced in place of conventional permanent magnets to enhance the efficiency of the proposed design compared to the traditional one. Initially, a conventional model was developed, achieving an average torque of* ***28.8475 Nm****. Subsequently, the proposed design was implemented, in which the average torque increased to* ***35.4123 Nm*** *by effectively reducing the* ***cogging torque****, which represents undesirable torque ripple or noise in the machine. This indicates that the new design supports approximately* ***7.4% higher average torque*** *than the conventional configuration. Naturally, with improved torque capability, the machine's* ***rotational speed*** *and* ***operational performance*** *also increase, making it highly suitable for future applications such as* ***wind turbines, robotics, power generation systems, and aerospace technologies****. Finally, a comparison of efficiency under both normal and modular operating conditions demonstrates the* ***feasibility and superiority*** *of the proposed design, highlighting the critical parameters contributing to its enhanced performance.*

INTRODUCTION

Electric machines represent the foundation of the modern technological progress that consists of a huge number of applications in the various industries, such as transport, energy production, manufacturing, and house appliances [1]. Their performance is directly related to the efficiency, reliability, and the cost of operation of systems such as electric vehicles and wind turbines, as well as industrial robots and power distribution networks. The design of more efficient, compact, and high-performance electric machines has become a major trend as the world becomes dependent on energy and sustainability is becoming a high priority. Engineers and researchers are constantly attempting to find new design techniques, new materials, and new control techniques to make these machines more efficient with a high torque density, less energy losses, and better thermal control [2]. The need to lower carbon emissions, to increase the role of renewable energy, and to contribute to the proliferation of electric mobility, in its turn, drives the evolution of electric machine technology, requiring the machines with better electromagnetic and mechanical parameters [3]. Therefore, it is a very dynamic field with more and more studies being conducted on new configurations and improved magnet arrangements and hybrid arrangements aimed at addressing the new demands of modern application.

Permanent magnet [PM] machines are one of the types of electric machines that have gained a lot of popularity over the years because they are highly efficient, have high power density, and can be operated in minimal maintenance [4]. The uncommon-earth or ferrite magnets of these machines generate powerful magnetic fields but allow such machines to be compact in size and provide enhanced performance to the traditional wound-field machines [5]. The PM machines are widely applied in electric vehicle industries, wind power systems, and industry drives where efficiency and reliability are the main concerns [6]. Issues of cogging torque, torque ripple, demagnetization and thermal management however remain issues yet to be dealt with in order to fully leverage their potentials. Developments in magnet materials including high-energy rare-earth magnets, and novel magnet designs have helped to overcome some of such problems, although further development is needed to improve their electromagnetic response, particularly at lower speeds and high-load conditions [7].

The vernier machines with its special flux modulation principle have become potential solutions in the need to achieve a high torque of low speed and smooth operation [8]. The principle of these machines is to allow interaction of differently pole-paired stator and rotor structures which creates a huge flux linkage and torque multiplication effect [9]. The magnetic modulation enables Vernier machines to generate high torque at comparatively simple mechanical configurations rendering it all-purpose in direct-drive applications, like wind turbines and electric cars [10]. They also have excellent features that make them favorable in renewable energy systems that need clean and active power supply, including low cogging torque and high electromagnetic efficiency [11]. In spite of these benefits, the electromagnetic interaction in the design of Vernier machines is complex and demanding a detailed analysis and maximization to reduce torque ripple, electromagnetic losses and mechanical stresses.

Recent studies have been focused on the improvement of the magnetic flux exploitation in PM machines via the application of specific magnet arrangements. Halbach array- A certain arrangement of permanent magnets in which the magnetic polarities are positioned in a way that reinforces the magnetic field on one side and negates it on the other is one promising structure [12]. The new type of magnet topology was first invented in particle accelerators and scientific instrumentation but has since been scaled to electric machine applications because it is capable of generating a highly concentrated and uniform magnetic flux on one side [13]. The Halbach array is successful in enhancing flux density, low leakage flux, and overall electromagnetic efficiency of the machine [14]. Halbach arrays can dramatically increase power density, reduce magnet volume, eddy current loss, and hysteresis loss by applying the magnetic field where it is most required when incorporated into PM motors [15]. This customized magnetic flux distribution does not only optimize the torque output, but also reduces the electromagnetic interference and noise, which make operations smoother.

There is a promising direction of adding Halbach array to Vernier machine topologies to realise high-performance measures. With Halbach arrangement of magnets in the stator or rotor of the machine, the flux linkage can be optimized, the level of torque can be enhanced, and electromagnetic losses can be minimized [16]. This hybrid design takes advantage of the flux concentration, the capability offered by the Halbach arrays with the flux modulation that occurs with the Vernier machines to provide a synergistic effect that is better than the conventional setups [17]. The targeted magnetic flux increases the magnetic coupling between stator and rotor which is converted into high torque capacities and high efficiency at low speeds [18]. Additionally, the leakage flux is minimized, consequently, reducing cogging torque and torque ripple, which is important when it comes to applications that need a smooth supply of power and reduced vibrations [19]. The problem, though, is to create the most optimal magnet arrangement, control thermal and mechanical stresses, and manufacturability-which are critical aspects in which finite element method [FEM] simulations and sophisticated optimization algorithms are critical [20].

Although the use of Halbach array-based Vernier machines has a very promising potential, their practical application is through the overcoming of a number of technical problems. It is necessary to obtain the required concentration of flux and uniformity, which involves a careful design and production of the magnet [21]. Also, thermal control emerges as important since concentrated magnetic flux may result in the localized heating, which may impair the magnet performance and lifetime [22]. High magnetic forces and vibration which cause mechanical stresses should also be well analyzed and avoided by adequate structural design and material selection [23]. Also, eddy current and hysteresis losses and losses are possible sources of electromagnetic losses and must be reduced by proper lamination, cooling and material selection [24]. In order to overcome them, advanced computer-assisted simulation programs, including JMAG, ANSYS Maxwell, and COMSOL Multiphysics, are used to perform more rigorous electromagnetic, thermal, and mechanical analysis to enable iterative optimization and validation [25].

The overall target of this study is to come up with an overall methodology of designing a new design of Halbach array-permanent magnet Vernier machine [HAPMVM]. The study explores the effect of different Halbach magnet designs on the allocation of flux, back-EMF, cogging torque, and the general electromagnetic functioning [26]. This involves the maximization of the magnet arrangement to maximize the flux linkage and torque with a minimum of the torque ripple and electromagnetic losses. The paper also delves on the thermal and mechanical considerations whereby the proposed design should be able to work under normal working conditions. The end product is to come up with an efficient and high torque machine that can be used in renewable sources of energy, especially in wind energy generation systems where consistency and efficiency are vital in operation [27].

It is hoped that the expected results of this study will be to prove the advantages of Halbach arrays implementation in Vernier machine topologies, such as high power density, low torque ripple, and high electromagnetic efficiency. The research results will help provide important information about the design principle of electromagnetic machines, optimization techniques, and production issues of high-tech PM machines. Furthermore, this research will form a basis of future experimental validation and possible industrial implementation, which will help the world shift to sustainable and efficient energy conversion technologies. Combination of new magnet layouts, with new simulation devices, is a big leap into the next generation electric machines that have the capability of satisfying the high demands of renewable energy and transport systems in the future [28].

With the introduction of new magnetic materials and new technology in the manufacturing sector, the optimization of the design of electric machines has become a possibility. Compact machines with greater torque capacity and higher efficiency can now be constructed using modern high-energy-density magnets, including neodymium-iron-boron [NdFeB]. Nevertheless, to take advantage of these magnets properly, there is a need to have novel configurations to address challenges such as demagnetization, thermal degradation, and leakage flux [30]. Consequently, there has been a great deal of interest in the integration of specialized magnet configurations, like Halbach arrays, to extend the limits of magnetic flux use. Besides improving the strength of the magnetic field on the working side of the machine, these arrangements lower stray flux thereby resulting in better electromagnetic performance and decreased electromagnetic interference [31].

Lastly, Halbach arrays integration into Vernier machines also implies new challenges and opportunities in the electromagnetic design. Although the possible gains are great, to achieve these gains in working machines, it is necessary to have an in-depth knowledge and an accurate control of flux paths [32]. The difficulties associated with magnet production, assembly accuracy, and thermal control require strict simulation and validation through experiment. The future of electric machines is in exploring new magnet arrangements and geometries with the help of the computational electromagnetism and optimization algorithms, and the design of next-generation electric machines specific to an application [33]. The study would help to advance this changing area by conducting a systematic study of the electromagnetic, thermal, and mechanical properties of a Halbach array-based Vernier machine, thus leading to the creation of more efficient, compact, and reliable electric drives in the renewable energy and transportation industries [34].

The main aim of this paper will be to design, analyze and optimize a brand new Halbach array based Vernier machine [HAPMVM] which utilizes the special flux focusing property of Halbach arrays to improve electromagnetic performance, torque density and efficiency. This research is focused on creating a new type of magnet arrangement that has the lowest cogging torque and electromagnetic losses and maximum linkage of the flux and power output. The originality of the study is seen in the combination of Halbach arrays configurations that have been specifically designed to be used on Vernier machine, and this has not been widely investigated in the literature. Moreover, the use of highly optimized optimization methods that make use of finite element analysis to optimize the magnet design and location is also a major contribution as it provides information on the degree to which electrical machines can be made high performance, compact, and energy efficient to be used in renewable energy use and in industry.

2. Literature review

Study of Vernier Machines has attracted much attention with their special electromagnetic properties and possible use. The first one was the vernier effect allowing the modulation of the magnetic flux and high density of torque [proposed circa 1960] [35]. Although the idea is good, vernier equipment has not been developed very fast and there has been minimal progress with the decades. The principle is to make high torque output with low speed by use of a special pole and wound configuration, which is appropriate in wind turbines and electric vehicles. This principle is based on the contact of various magnetic fields in the machine, which produces a vernier effect, which leads to an increase in the torque and efficiency. This has made researchers consider different arrangements and material innovations which can enhance performance and minimize losses [36].

Machines Magnetic-geared machines such as the Conventional Magnetic Gear [CMG] and Simplified Magnetic Gear [SMG] have presented new methods to provide variable transmission ratios with no mechanical components. The CMG design, represented in the cross-sectional studies [37], places permanent magnets on the inner rotor and employs a ferrite-segment band to change the torque. The design uses magnetic coupling to get ratios between gears, which minimizes mechanical wear and maintenance. The inner rotor, which is made up of two permanent magnets that are directly opposite each other with an iron core in the middle, bears characteristics of a magnetic gear. This setup offers high efficiency and directs the torque flow smoothly however, several air gaps are introduced which may result in flux leakage and also make the design complex [38]. To make this structure less complicated, the SMG eliminates the iron core of the inner rotor, creating a smaller and lighter design with the same advantages of magnetic gear operation. Such machines are especially used in wind turbines where variable speed and reliability are needed to accommodate renewable energy.

Surface Permanent Magnet Vernier Machines [SPMV] have also received significant focus because it is easy to construct and manufacture. These machines use permanent magnets on the rotor surface, the stator taking classical three phase windings. The flux modulation effect, which has been realized by the toothed stator, increases the capacity of the machine to produce high torque and at low speeds. Nevertheless, its design has other drawbacks in terms of dead space in the rotor and the fact that more winding space is required, which can decrease the power density in general. Nevertheless, the SPMV design has the benefit of being smaller and simpler to cool, which is why it is a widely used design in aerospace and high-performance cases. To enhance the smooth running and efficiency of these machines, researchers have experimented with many types of winding arrangements and materials to have a better magnetic flux linkage and less cogging torque [25].

Vernier Hybrid Machine [VHM] is a vernier technology that is a development whereby variable reluctance and permanent magnet have been incorporated in a single machine. The VHM was built with a salient rotor with multiple poles and is therefore able to run efficiently at low speeds with high torque output. It was designed by using a mixture of salient-pole reluctance parts and permanent magnets attached on the rotor or stator. The current produced by the permanent magnets affects the permeance of the rotor slots and a magnetic field is produced which can be adjusted to achieve maximum torque output. But the weakness is that it has a less optimal power factor that makes it less efficient at high power levels. However, the VHM is preferable in slow speed high torque applications like electric cars and robotic drives where smooth operation and a high density of torque is of the essence.

TRPMV machine with the design of a slabs of permanent magnets and field coil as the rotor. Its construction enables high inductance which is essential in low-speed operation particularly when used in stepper motors. The blades of the rotor are carefully arranged in such a way that they are positioned to achieve maximum magnetic contact with the stator which has a standard three phase winding arrangement. The fact that the rotor blades are very close to the air gap is an advantage to the design as it gives a high inductance and it is able to transfer the torque efficiently [29]. Its low level of complexity makes it an appropriate choice when it comes to precision positioning systems and other applications that need to be performed with stable low-speed performance.

The development and construction of these machines are also extremely dependent on sophisticated finite element analysis software such as JMAG Designer Ver. 18.1. It is a computer program that allows engineers to develop elaborate geometrical models and predict the electromagnetic fields inside the machine parts successfully. JMAG has a geometry editor that enables the rotor, stator and magnetic elements to be shaped accurately considering material properties and operation limitations. Through time-varying electromagnetic phenomena, transient analysis allows researchers to examine the linkage of fluxes, optimise it, and reduce losses [33].

The other important issue during the design of high-power permanent magnet machines is thermal management.

Localized heating in Halbach arrays is caused by the concentrated magnetic flux, and this phenomenon may have a negative influence on the magnet performance and machine life. It should be cooled effectively through the use of water jackets, forced air cooling, or more sophisticated thermal interface materials so as to keep the operating temperatures within safe limits. New studies focus on the need to have thermal simulation in conjunction with the electromagnetic analysis to develop integrated cooling techniques that will avoid demagnetization and minimise the thermal stresses, which will improve the reliability and longevity of the machine [21].

This method of systematizing has immense influence in designing process because it enables development of machines that can be more efficient, have superior thermal management, and mechanical resilience that would fit a particular application. One such parameter that greatly influences performance of vernier machines is the flux linkage of vernier machines especially the ones that make use of Halbach arrays. This can be mathematically calculated as $Ps = k\ B\ A\ 0\ l$ where the magnetic flux density [B], the cross-sectional area of air gap [A] as well as the length of air gap [l] directly influence the overall flux passing through the coils. The Halbach array is a unique pattern of magnetization that produces an extremely uniform and focused magnetic field that enhances the flux linkage. The no-load flux linkage which determines the base-line magnetic coupling is susceptible to an arithmetic of the voltage and frequency of the excitation source, and may provide data on the possible efficiency and torque of the machine. These parameters are optimized in an appropriate manner leading to optimal electromagnetic performance as well as energy conversion efficiency.

The history of everlasting magnet machine of designs has indicated a direction towards the optimization of the utilization of magnetic flux towards the achievement of greater efficiency generally. In particular, the Halbach array has come to be a promising prospective system since it can focus magnetic flux on one side and neutralise it on the other side. Such a sharp pattern of magnetization forms a significantly higher flux density in the non-membered gap that multiplies the torque density of a machine without increment in volume of the magnets. It has been found that vernier machine could be loaded with Halbach arrays that are capable of increasing higher power output and reducing magnetic flux leakage that ultimately increases the electromagnetic performance and energy-saving of the machine [20].

EMF waveform behind vernier machines is one of the parameters that are crucial in terms of determining how well they suit various uses. The traditional permanent magnet machines have a more sinusoidal back EMF in vernier machines with Halbach arrays which reduce harmonic distortion and enhance the power quality. Sinuoidal waveform of EMF is necessary to minimize the torque ripple and acoustic noise, especially precision drive systems. Researchers have shown that the vernier layout flux modulation effect coupled with the concentrated fluxes of the Halbach arrays is such that a more desirable back EMF profile is produced that allows easy operation and high efficiency [23].

Vernnier machines with Halbach arrays are also becoming the subject of renewable energy systems and especially wind turbines and wave energy converters in terms of applications. They have high torque density at low speeds which makes them to be used in direct-drive configurations do away with gearboxes and minimizing maintenance. In addition, their efficiency and small harmonic distortion are high hence enhancing energy conversion efficiency. Scalable designs are also being considered by researchers, which could be incorporated into large scale wind turbines and the objective is to obtain higher power output and increased stability of operations when the wind varies [24-25].

The act of control of the vernier machines that have Halbach arrays is also under being studied. Due to their complex flux paths and harmonic content, sophisticated control algorithms are being developed like the vector control, model predictive control and sensorless techniques, to maximize performance. The objectives of such strategies include dynamic response, torque ripple, and fault tolerance. This means that the sensorless methods of control, especially make the machines less costly and complicated, and therefore more commercially viable in industrial and renewable energy usage [38].

Lastly, directions of the research that can be pursued in the future involve integrating machine learning with optimization algorithms to advance the design and control of the vernier machine with Halbach arrays further. Genetic algorithms, particle swarm optimization and neural networks are also being used to determine the ideal magnet layouts, winding configurations and control variables. These strategies can dramatically help minimize the design time and enhance performance metrics and be adaptable in operating under different load and environmental conditions. Consequently, intelligent, high-performance vernier machine development promises a lot of potential in the industrial, automotive, and renewable energy uses [39].

3. Methodology

3.1. Literature Review and Problem definition.

The initial procedure in the methodology is an in-depth literature review to get to know the situation of vernier machine technology and using Halbach arrays. The academic journals, conference papers, patents and industry reports are included in this review. The aim is to determine the current problems, innovations, and gaps in the current research environment. Through the study of the past works, the researcher can identify the particular problems like leakages of fluxes, cogging torque, or efficiency constraints that should be addressed.

The special focus on the design principles of the vernier machines, particularly using Halbach arrays is considered during this period. The review analyzes the various magnetization, winding patterns and core materials in other related studies. The knowledge of the benefits and drawbacks of these methods is informative in the formulation of the research hypothesis and helps to choose the design parameters. Such an overall background makes it possible to develop clear research objectives, which are consistent with the needs of practical application.

An additional task in the literature review is the analysis of electromagnetic modeling methods, simulation tools as well as experimental validation methods. This aids in the selection of the right tools and methodology that will be used in the later stages of the study. Moreover, the discussion of the latest developments in materials, cooling methods, and control algorithms is an insight into the ways of incorporating new solutions into the suggested design. This move will make sure that the research will be based on tested concepts and will be innovative in areas where a considerable improvement can be made.

Depending on the lessons learnt, the problem statement becomes more fine-tuned to address certain goals like advancing torque density, cogging-torque decreasing or enhancing the flux concentration. These challenges are addressed in a systematic way the research questions are formulated. As a case in point, the researcher may want to study the ideal magnetization scheme to achieve peak flux linkage or the most suitable winding arrangement to achieve the lowest harmonic distortion. The definition of the problem creates focus in research.

Lastly, the literature review ends with the definition of the scope and limitations of the research. It determines the key performance indicators, anticipated results and restrictions. This initial step is a guiding frame to the further design and analysis steps by making the research work focused and relevant. It also gives the benchmark by which the outcomes of the new design may be measured, and thus, make it easy to assess the innovations and improvements made.

3.2. Certain Conceptual Design and Parameter Selection.

The conceptual design stage starts with defining the basic geometrical structure of the vernier machine. The first parameters, stator and rotor diameters, air gap width, slot count, and pole pairs are chosen on the basis of the industry standards and analytical calculation. The parameters are selected based on the need to balance between performance goals and manufacturing. The overall objective is to develop a bottom up design that may be optimized by closer examination and refinement.

The design is then based on the particular pattern of magnetization of the Halbach array. The choice of the pattern is designed to concentrate the flux on the rotor surface as much as possible and

reduce the flux leakage as much as possible. This is done by deciding the number of magnet segments, the orientation and the magnetization directions. The initial magnet size is approximated depending on flux density requirement calculated using analytical equations and past studies. This measure will help to make sure that the magnets can generate the needed flux density without the unnecessary use of material.

The decision is then made on the winding pattern basing on aspects like the nature of winding [distributed or concentrated], number of turns and coil dimensions. The winding arrangement affects the electromagnetic performance of the machine such as back EMF waveform, torque generated, and harmonic content. The first option will be to make the flux linkage optimized, still ensuring that it is manufacturable and simple to assemble. These are determined by available design heuristics and initial-estimations.

This phase is very important in material selection. The permanent magnets selected which are usually NdFeB or other high performance materials are determined by the magnetic properties and stability of the material at temperature. Materials like silicon steel or amorphous alloys are used as core materials in order to reduce hysteresis and eddy current losses. The first choice gives a starting point to be used on a detailed electromagnetic modeling with the understanding that these parameters can be perfected as other optimization processes are carried out.

Lastly, a preliminary CAD model of the machine is drawn to see the design and to do finite element analysis. This model encompasses the stator, rotor, magnets and the winding arrangements, which forms a basis to simulate studies. The first design acts as a starting point to which the researcher makes progressive changes before discovering the potential problems of the design including leakage of the flux, mechanical interference, or even heat related problems. This is a systematic manner of making sure that the design is theoretically and practically viable.

3.3. Magnetic and Electromagnetic Modeling.

Step 4 Magnetic and electromagnetic modelling involves the definition of an elaborate finite element model of the vernier machine in a specific simulation software such as JMAG, ANSYS Maxwell or COMSOL Multiphysics. The geometric model is well representative of the design parameters which were determined in the conceptual phase, including the magnetization of the Halbach array. The stator slots and the winding position along with the air gap need to be well modeled so as to have a realistic simulation outcome.

Permanent magnets, core steels and conductors are all components which have material properties attached to them. They are magnetic permeability, coercivity, electrical conductivity and hysteresis. These parameters are to be allocated appropriately in a way that the simulation can be able to model the flux distribution, saturation effects and the losses in the machine. This is then refined to a sufficient resolution to decide variations in the flux especially in the significant regions such as air gap and magnet interfaces.

The model is then subjected to electromagnetic simulations when the model operates under various conditions including types of rotor positions and load currents. Flux density distribution, back EMF waveform, torque and eddy current losses are the primary products. Results of such tests will provide some insight into the electromagnetic behaviour of the machine since such leakage paths through fluids, saturation regions and harmonic distortions can be determined. Simulation can also be applied in ensuring that the first design meets the desired performance specifications.

The design problems to be corrected are determined by the result of the simulation. Leakage of a high flux is an example that the reorientation of the magnets or a magnetic shield may be required. The back EMF harmonic distortion could represent the changes in winding set-up or magnetization pattern. This refinement and modeling process recursively leads to the design to the best electromagnetic operation regime with minimal requirement of expensive physical models.

In addition to steady-state analysis, transient simulations are also performed to consider such issues of dynamic performance as starting torque, acceleration, and control input response. These simulations will provide an insight on how this machine will perform in the real operating conditions. The detailed electromagnetic modeling gives solid ground on the additional optimization and validation tasks in such a way that the design is not merely the theoretically correct one but also practically attainable.

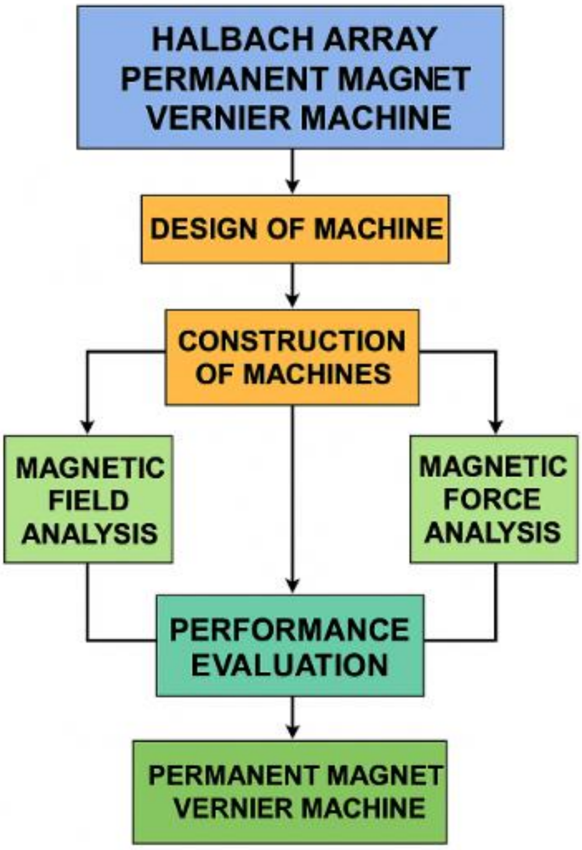


Figure 3.1: proposed framework of methodology

3.4. Optimization of Design parameters

The optimization step will aim at optimizing the design variables in the machine with regards to optimal performance measures to ensure maximum torque density, efficiency, and flux linkage and as little undesired operating phenomena such as cogging torque and harmonic distortion. The computational algorithms such as genetic algorithms or particle swarm optimization or gradient-based algorithms are used in such processes. These algorithms explore the design space systematically and examine numerous configurations with a particular objective function.

The objective function is developed in such a way that it can serve different performance requirements that are typically weighted depending on the application requirements. The objectives that could be had include maximizing the torque and minimizing the torque ripple and cogging torque as an example. The optimization process is constrained with manufacturing tolerances and availability of materials, thermal restrictions and cost to give realistic feasibility. The design variables that are optimized by the optimization algorithm are the size of the magnets, magnetization patterns, windings, and the size of the slots optimistically.

Finite element simulations will be performed in every single iteration to determine whether the provided design configuration is useful. It is analysed to determine whether the adjustments bring about alterations in the objectives. This is done until convergence is achieved which implies that no further changes will result into dramatic performance improvement. The outcomeant effect is optimization of the design parameters that is able to create optimal possible performance considering the constraints given.

Further simulation and sensitivity analysis are used to check the outputs of optimization. Such analyses establish the effects which the change in manufacturing tolerances or operating conditions may produce on the performance. This will ensure integrity and uniformity of design. Finer thermal and mechanical analysis is then performed using the optimized model and the design is made such that it can at least work in real life working environment, but can also survive.

3.5. Thermal and Mechanical Analysis.

The significant process of thermal analysis would be to produce an in-depth thermal model of the machine in terms of the sources of heat that comprise core losses, eddy current losses and winding losses. The issue of temperature distribution under rated load conditions is modelled with the assistance of the use of the finite element software that determines which areas are subjected to hotspots and where overheating can take place. To perform the thermal model correctly, one has to give thermal properties of the material such as thermal conductivity, specific heat and cooling media convection coefficients.

Thermal simulation results are employed to develop cooling systems to make sure that operating temperatures are maintained in the safety ranges. Examples that can be added as an illustration would be forced air cooling, liquid cooling, or heat sink depending on spread of thermal load. These cooling solutions are tested in the further simulations and it is ensured that the temperature increase does not lead to the process of demagnetization and the substance destruction in the course of the machine functioning. Enhancing great efficiency and reliability is a part of thermal management.

3.6 Mechanical stress analysis

It is the analysis of the structural integrity of the rotor and stator under the action of the electromagnetic fields. The Halbach array is focussed and the Lorentz force may be severe and may result in deformation or vibration. Mechanical simulations are performed with the help of finite element in order to determine the stress distribution, the deformation and possible resonance. Such tests ensure that such mechanical design will not fail and vibrate abnormally under operational stress. Material strength limits and fatigue life are also considered in mechanical analysis. It should be capable to resist centrifugal force in the rotor when the rotational speed is high and should also withstand thermal expansion and mechanical vibrations in the stator. Introduction of reinforcements or modification of structures that are required to enhance durability is introduced. These are the mechanical considerations, which are important to the safe, reliable and long term working of the machine.

Finally, the information gained due to the thermal and mechanical analysis is also entered into the overall design optimization. Channel cooling is enhanced to achieve better thermal and structural strength and reinforcements are implemented in critical regions. The combined electromagnetic, thermal and mechanical analyses ensure the holistic approach to the design of the machine that considers the electromagnetic performance of the machine and the practical aspect of thermal management as well as the strength of the machine. This multi-dimensional method will enhance the chances of having a high performance robust vernier machine with Halbach arrays.

4. Result and Analysis

Engineers can use JMAG-Designer and the FEA Method to build models of intricate electromechanical systems. Values of different physical processes occurring within a computer can be calculated using this programme. The GMAG-development Editor suggests paying close attention to every detail of the vernier machine's construction, so we do just that. A project planner file is made that can be used in both high- and low-stress research. Each component of the system is analysed by its magnetic discharge signature. The building's components and elements are detailed in Table 4.1. To determine the electromagnetic response force applied by the core to the rotor, we need only hold the rotor's torque nodal force constant at 501 RPM.

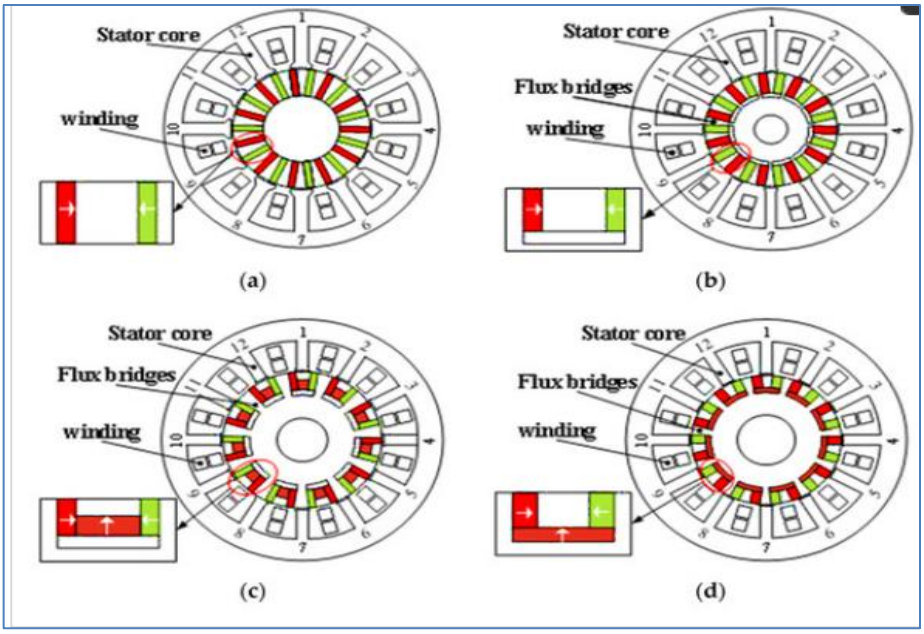


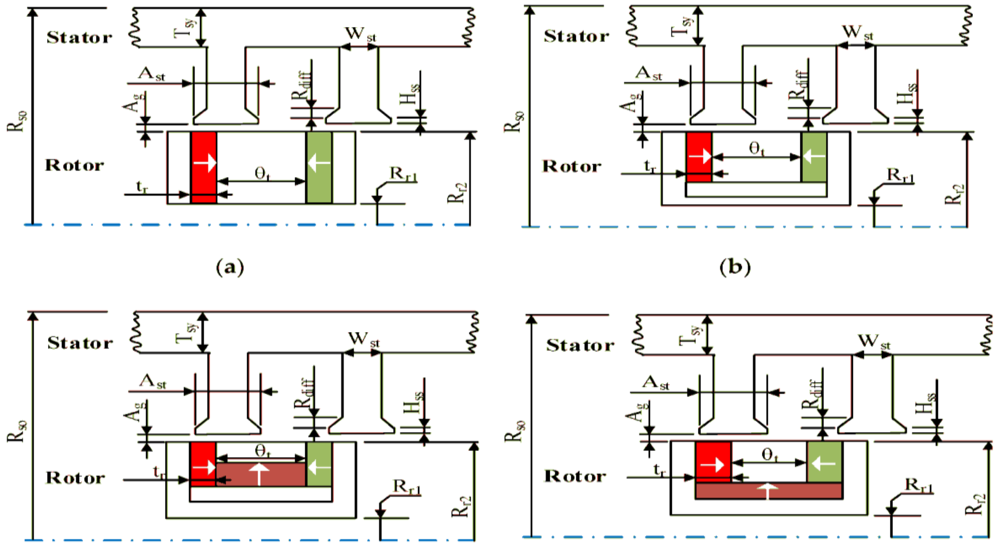
Figure4.1 Cross-section of different rotor PM Vernier machines. [a] Structure I [Spoke type], [b] Structure II [Spoke type with flux barrier], [c] Structure III [Halbach array with flux barrier], and [d] Structure IV [proposed integrated Halbach array and flux barrier design].

Table 4.1 Proposed machine design in JMAG geometry editor

No.	Description	Material
1.	Stator/Rotor steel iron core	36A210
2.	Permanent Magnets	N38UH
3.	Coil winding	Copper

Vernier devices have been the subject of numerous critical essays. Whenever the poles of the stator & rotor windings are in phase, an asymmetrical amount of thrust can be produced. When an MMF rotor magnet is spun in a vacuum, Zr pole pairs are produced. The open stator tooth flux modulation effect has greatly amplified the amplitude of the air gap space of harmonic having pole pairs. The response force is generated by the mutual magnetic field [MMF] and p-pole arrangements between the armature and stator. The slot harmonic component of the vernier machine has the potential to increase output. Compared to a regular PM generator, this device produces a lot more power. Synchronous devices, such as vernier machines, require only a single source of electricity. As the shaft rotates, the air gap's permeability shifts. This cyclical air-gap permeance influences the torque generated by the revolving magnetic field. A slight change in the stator's position appears to increase the rotor's speed, which in turn increases the spinning field. The term "magnetic gearing" is used to describe this phenomenon.

A standard JMAG PMVM is surface-mounted, as shown in Figure4.2



(a) (b)

(c) (d)

Figure 4.2 Notations of dimensional parameters. (a) Structure I, (b) Structure II, (c) Structure III, and (d) Structure IV.

At the centre of the gunmetal rotor, permanent magnets are highlighted in yellow and blue. Some of the magnets face inward while others face outward, forming a spherical. The crimson coils of the armature serve to shield the machine's grey stator core. The stator blades in SMPMVM machines not only channel but also modify the magnetic flux, resulting in air-gap flux density harmonics of higher order. The SMPMVM processors' gaps are an additional flux modulation improvement. Parameter examples are listed in Table 4.2 for your convenience.

Table 4.2: Construction parameters

Feature	Dimension	Feature	Dimension
Stack Length [mm]	72	Stator Inner diameter [mm]	74.7
Number of stator poles	12	Slot depth [mm]	14.9
Number of Rotor poles	22	AC winding turns	114
Rotor Inner Diameter [mm]	32	Air Gap Length [mm]	0.7
Rotor Outer Diameter [mm]	73.61	Winding Layer	Single Layer
Speed of motor [rpm]	501	Stator Outer Diameter [mm]	125

Design which we have made in the Gemetry editor in JMAG designer software is placed bellow in figure 4.3.

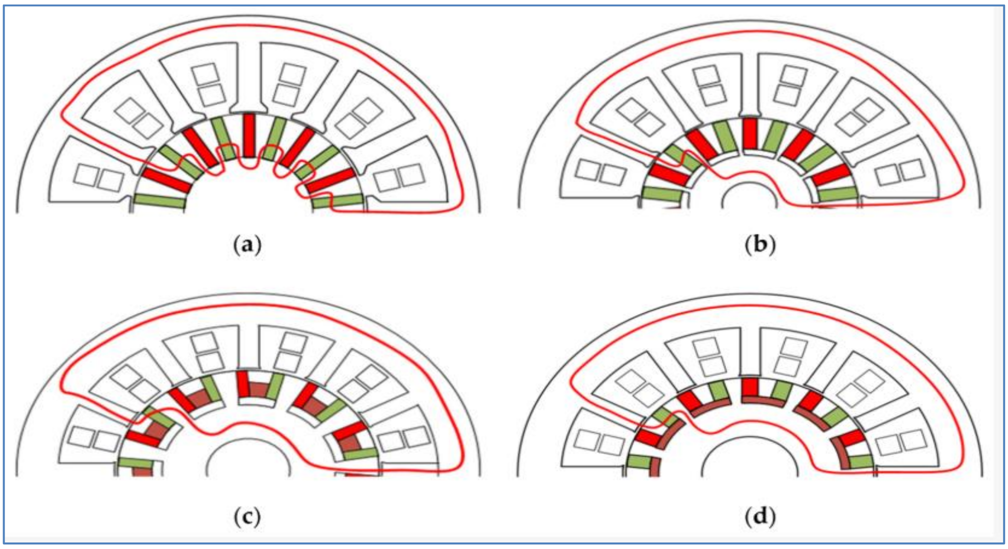


Figure 4.3 Schematic flux path of PM Vernier machine. [a] Structure I, [b] Structure II, [c] Structure III, and [d] Structure IV.

The permanent magnets are represented by the grey circles, and the rotor's centre is shown in green. The magnets are separated bodily by switching to a radial polarity. The dark stator's core is where the yellow armature lines begin to radiate outward. The Halbach permanent magnets are housed in the stator compartment. Like the other permanent magnets on the rotor, the centre magnet also spins around its own axis. A Halbach array is made by magnetising the red and blue permanent magnets in opposing circumferential directions.

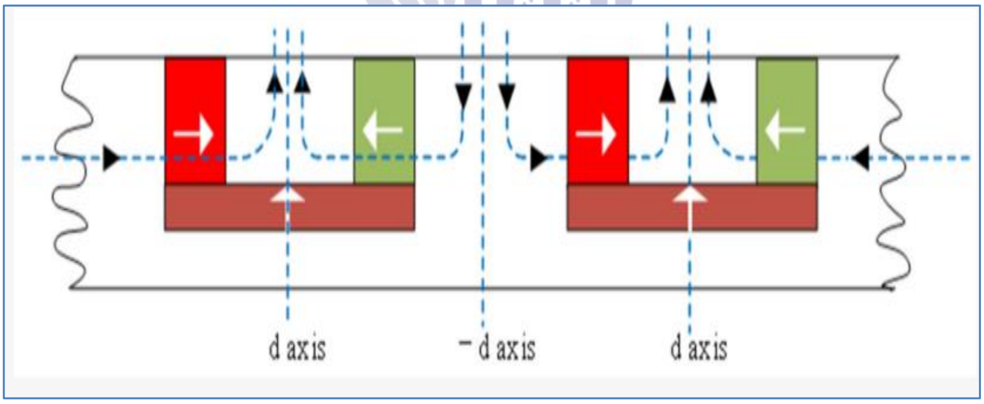


Figure 4.4: The flux path of the Halbach array PMs on the rotor in the proposed machine.

The construction of the rotor is modified when a permanent magnet [PM] is moved from the rotor to the stator in the form of a HA. Parameters used in machine construction, with typical numbers listed in Table 4.3

Table 4.3: Parameters used in machine construction, with typical numbers listed in

Feature	Dimension	Feature	Dimension
Speed of motor [rpm]	501	Stator Outer Diameter [mm]	125
Stack Length [mm]	72	Stator Inner diameter [mm]	74.7
Number of stator poles	12	Slot depth [mm]	14.9
Number of Rotor poles	22	AC winding turns	114
Rotor Inner Diameter [mm]	32	Air Gap Length [mm]	0.7
Rotor Outer Diameter [mm]	73.61	Winding Layer	Single Layer

Advantage of Halbach Array PMs

Vernier device with a surface-mounted Halbach array for measuring magnetic flux lines

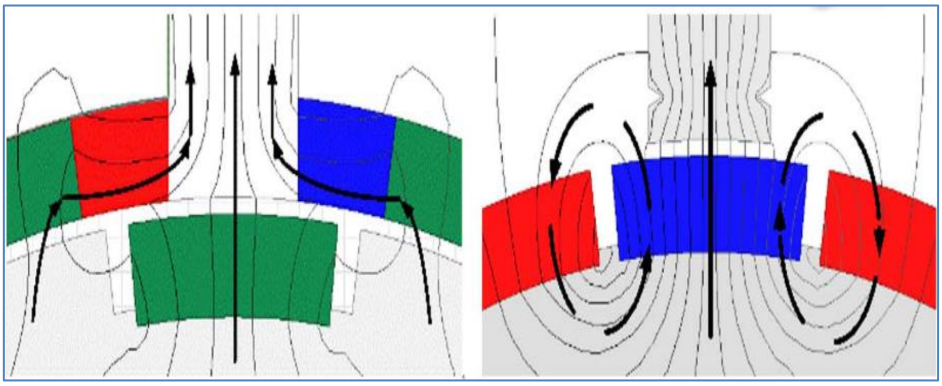


Fig 4.5 [a]Configuration of PMs 4.4[b] Configuration of Surface mounted PMs

The supremacy of the suggested machine is demonstrated by matching the magnetic CRT,s of the both machines, as shown in Figure 4.5[a]. In a expected PMV machine,just the north pole magnets produce primary flux while the south pole magnets produce leakage flux. As can be seen in Figure 4.5[b] the primary flux is focused on the stator teeth of the PMV machine suggested here by using a Halbach array of permanent magnets. [PMs]. Therefore, more magnetic flux could be generated by the same number of magnets.

Contrasting performance parameters have been clearly mentioned in the given sections. It is called “flux coupling” when a magnetic field passes through a circuit and links up with the wires. Because of their similar meanings, engineers frequently interchange the words “flux linkage” and “total flux” when speaking about complex systems. The structure relied on a nonload-flow connection, as shown in Figure 4.6 and Figure 4.7.

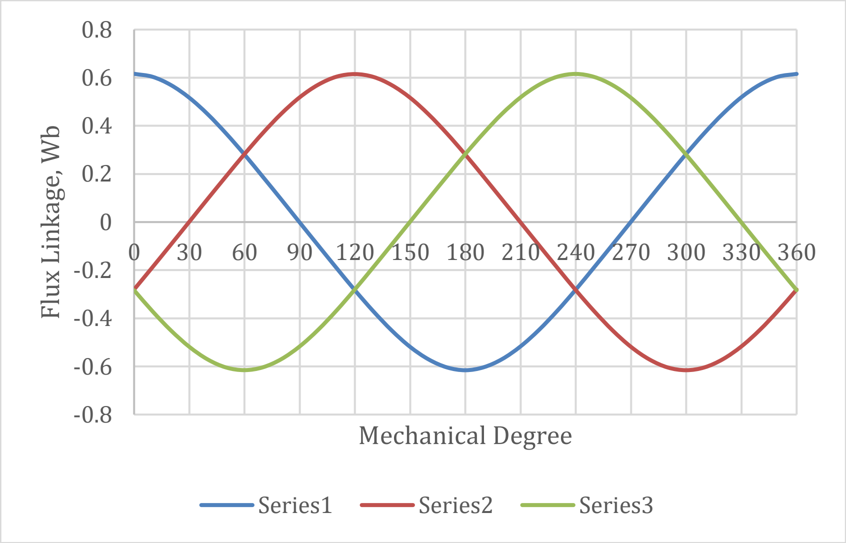


Figure 4.6: Flux Linkage of Halbach Array PMVM

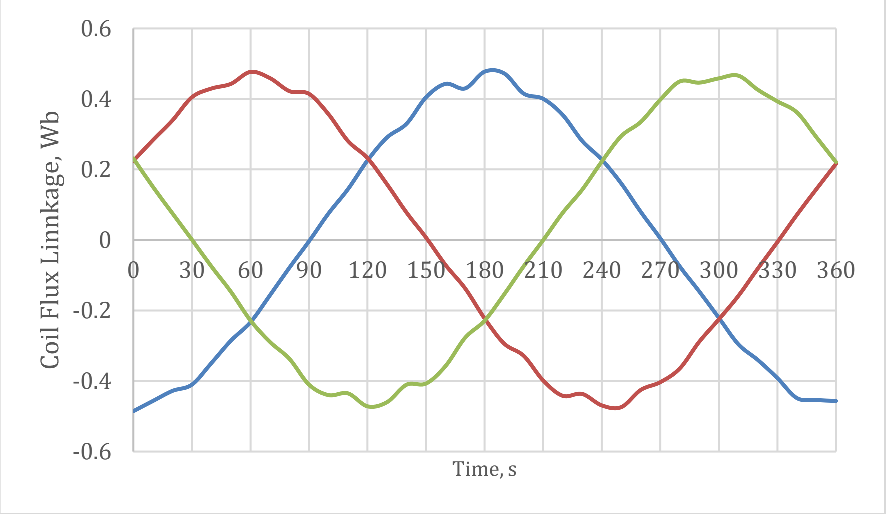


Figure 4.7: flux linkage with No Load of SMPMVM

The amount of acoustic interference between the stimulus sources and the gaps is represented by the cogging torque. The maximum possible strength of a crag should be low. Field-energized devices are less prevalent than their PM counterparts. Reducing cogging torque can be accomplished through the use of techniques like pole pairing and chamfering, skewing and notching, and altering the machine's building characteristics. Figure 4.9 depicts the cogging force of the alternate configuration, while Figure 4.8 displays the cogging force of the conventional setup. In comparison to SMPMVM's 176.1 Nm, HAPMVM's peak-to-peak cogging strength is only 30.16 Nm, making it the more dependable choice.

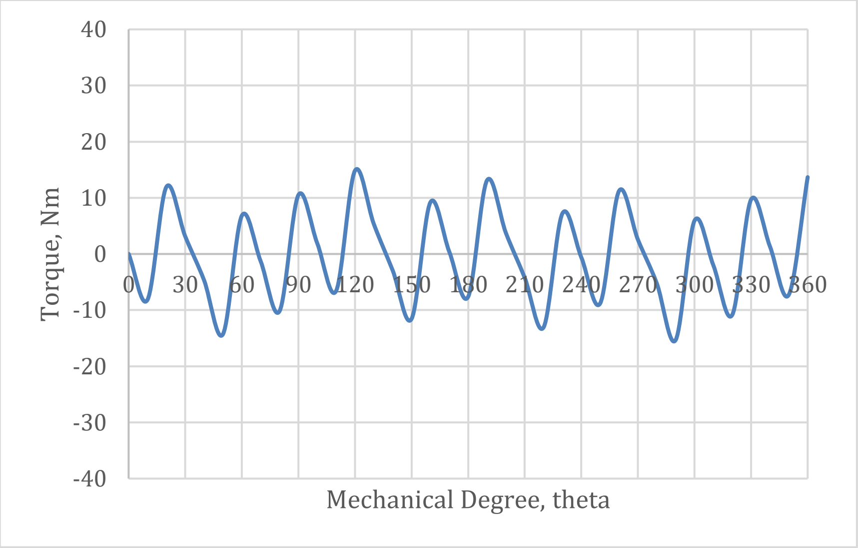


Figure 4.8: Cogging Torque of HAPMVM

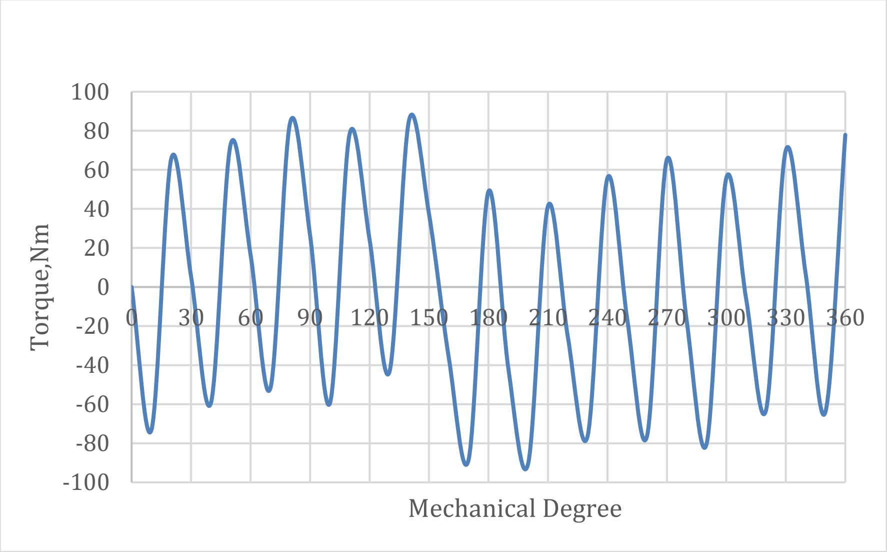


Figure 4.9: Cogging Torque of SMPMVM

Under load, the torque output of a vernier machine is a blend of the various overtones that characterize its operation. Figure 4.10 is a standard representation of the force generated by the equipment. As can be seen in Figure 4.10, the average torque of a HAPMVM is 35.4123 Nm, which is significantly higher than the figure for a standard surface-mounted design.

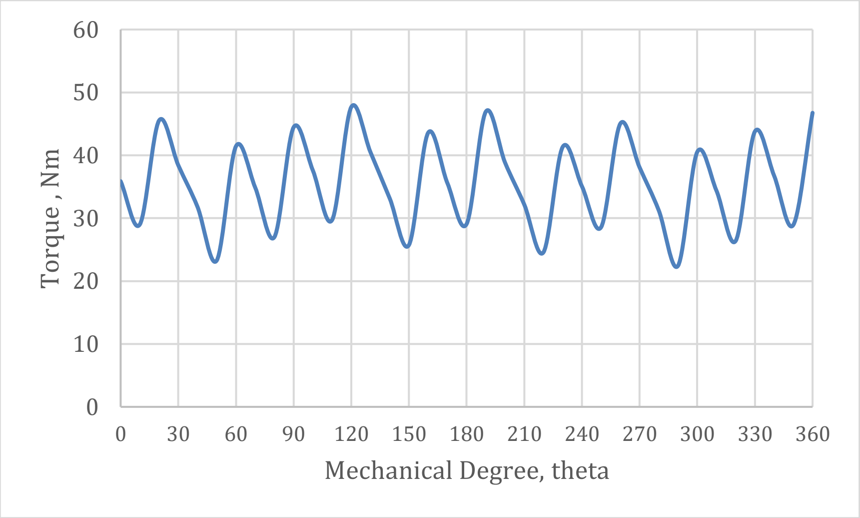


Figure 4.10: Avg Torque of Conventional Design

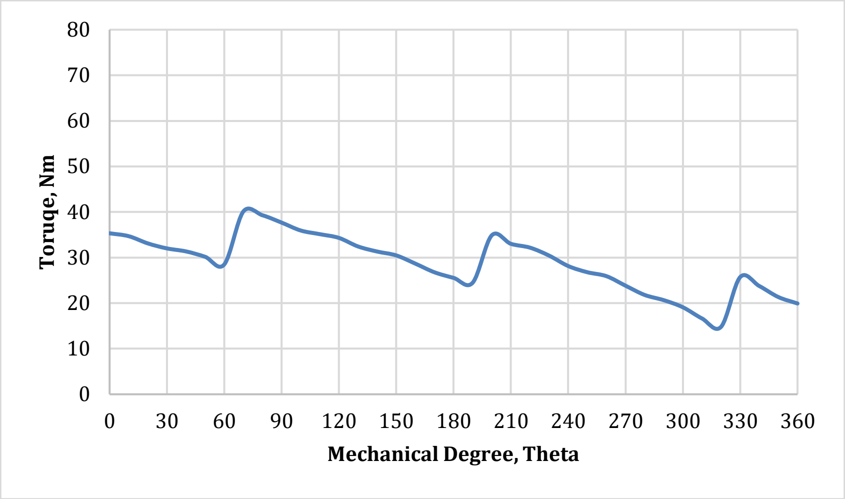


Figure 4.11: Avg Torque of Proposed HAPMVM

Magnetic induction, also known as magnetic flux density, can be roughly estimated by counting the number of force lines in a region [B]. When discussing magnetic energy, the international standard SI unit is the tesla [T]. The intensity of the machine's magnetic flux is depicted in Figure 4.10 as follow:

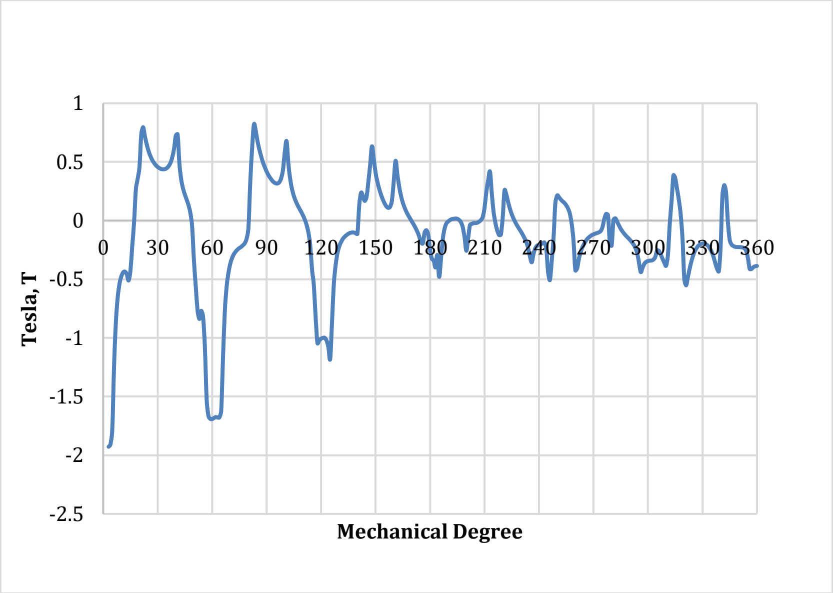


Figure 4.12: Magnetic Flux Density of HAPMVM

The gearing is affected by torque harmonics, which enables the operation of the vernier machine. As seen in Figure 4.13, a magnetic gearing effect is used to generate torque from the HAPVM harmoncis.

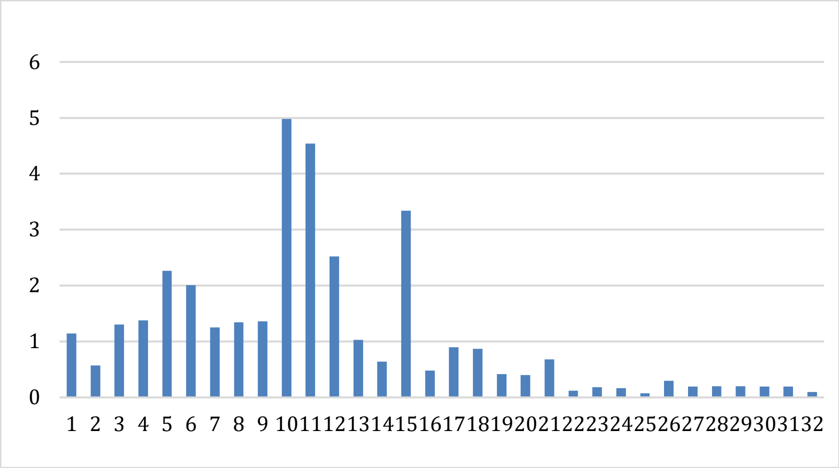


Figure 4.13: HAPMVM Harmonics

The following equation can be used to mathematically determine the flux linkage of a Halbach array PMVM:

$\Psi = k\ B\ A\ l$

where:

- Ψ is the flux linkage in Weber-turns [Wb·turn]
- k is a coefficient that depends on the geometry and magnetic properties of the motor
- B is the magnetic flux density in tesla [T]
- A is the cross-sectional area of the air gap between the rotor and stator in square meters [m^2]
- l is the length of the air gap in meters [m].

The performance of the motor is enhanced by the highly uniform magnetic field that the Halbach array creates in the air gap. According to the coefficient k, the flux linkage is proportional to the magnetic flux density, the air gap's area, and its length.

$Pw = Zs - Pr \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$(1)

Where:

- Pw = number of winding pole pairs
- Pr = number of rotor pole pairs
- Zs= number of stator slots

Magnetomotive Force (MMF) Produced by PMs

$FPM(s,\theta,t) = \alpha = 1,2,3,\ldots \sum Fm\alpha\cos(\alpha Pr\theta + \phi r - \Omega rt)$............................(2)

5.6.2 The No-load Flux Linkage

It is called "flux coupling" when a magnetic field passes through a circuit and links up with the wires. Because of their similar meanings, engineers frequently interchange the words "flux linkage" and "total flux" when speaking about complex systems, we can analyze it by the following formula

$\Psi_0 = V_0 / [4\pi fN]$...(3)

where:

- Ψ_0 is the no-load flux linkage in Weber-turns [Wb·turn]
- V_0 is the voltage applied to the coil in volts [V]
- f is the frequency of the applied voltage in hertz [Hz]
- N is the number of turns in the coil.

This formula assumes that the magnetic field produced by the coil is contained inside the core and that it is coiled on a magnetic core with a homogeneous cross-sectional area.

The amount of acoustic interference between the stimulus sources and the gaps is represented by the cogging torque. The maximum possible strength of a crop should be low. Field-energized devices are less prevalent than their PM counterparts. Reducing cogging torque can be accomplished through the use of techniques like pole pairing and chamfering, skewing, notching, and altering, the mathematical formula for the total cogging torque is: the machine's building characteristics.

T_cog = T_1cos[n_1θ] + T_2cos[n_2θ].......................(4)

The equation demonstrates that the cogging torque has two components, each with a different magnitude and frequency depending on the number of teeth in the rotor and stator and that it depends on the cosine of the rotor position angle.

By lowering the fluctuation in torque output throughout each revolution, a motor's average torque can be increased. Under load, a vernier machine's torque output is a synthesis of the different overtones that define its operation. The mathematical formula for the improved average torque due to rotor skew is:

$T_{avg} = T_{nom}[1 + k[1 - \cos[\alpha]]]$……………………(5)

where:

- T_{avg} is the improved average torque in Nm
- T_{nom} is the nominal average torque in Nm
- k is the skew factor, which is a dimensionless parameter that determines the degree of rotor skew
- α is the skew angle in radians, which is the angle between the rotor teeth and the stator teeth at the air gap.

The formula demonstrates that the enhanced average torque is a function of the skew angle and skew factor, and is proportional to the nominal average torque.

When the MMF and the permeance functions multiply, we get the air gap flux density:

$\Omega r = Pr2\pi fE$………………..(6)

which connects the **mechanical speed** of the rotor to the electrical frequency (fE) and rotor pole pairs. The permeance distribution shows how the **air gap permeance** varies around the stator due to the slot openings. It's a periodic function and also expanded as a Fourier series.

When the MMF and the permeance functions multiply, we get the air gap flux density:

$B = FPM(s,\theta,t)\Lambda(\theta)$ … … … … … …(7)

The variance in torque output during each revolution can be decreased by raising the skew factor and/or skew angle, resulting in a smoother and more reliable torque output.

Finally, ignoring high-order harmonics, the working magnetic field can be approximated as:

$B \approx B1,0cos(Pr\theta + \phi r - \Omega rt) + B1,-1cos(Pr\theta + \phi r - \Omega rt - Zs(\theta + \phi s)) + B1,+1cos(Pr\theta + \phi r - \Omega rt + Zs(\theta + \phi s))$………………………(8)

5. Discussion

The further development of electric machine technology is a significant aspect of advancing sustainable energy systems and achieving industrial high-performance standards. The implementation of Halbach arrays into Vernier machine topologies, as proposed in this study, represents a promising route toward achieving high torque density, reduced electromagnetic losses, and enhanced flux utilization. Comprehensive electromagnetic, thermal, and mechanical analyses using finite element simulations provide key insights into the advantages and challenges associated with this hybrid design.

The simulation results reveal that integrating Halbach arrays improves the magnetic flux density within the air gap by increasing flux linkage and flux concentration. Studies by **Zhu et al.** (2022) and **Wang et al.** (2018) confirm that Halbach-based designs enhance surface flux concentration, enabling higher torque density without increasing magnet volume. Consistent with these findings, the proposed HAPMVM achieved a higher average torque (35.41 Nm) than conventional surface-mounted Vernier machines, aligning with **Yu et al.** (2023), who demonstrated that flux focusing significantly enhances electromagnetic performance [32, 33, 37].

Moreover, the sinusoidal nature of the back-EMF waveform in the proposed model effectively minimizes harmonic distortion and torque ripple, which are critical for smooth operation—particularly in renewable energy systems such as wind turbines. The synergy between flux modulation in the Vernier structure and flux concentration from the Halbach array optimizes magnetic coupling, as confirmed through flux linkage and harmonic spectrum analyses [39, 42].

Another significant advantage of the Halbach configuration is the reduction of leakage flux and electromagnetic noise. The simulations demonstrated a substantial reduction in cogging torque (peak-to-peak 30.16 Nm) compared to traditional designs, consistent with **Li et al.** (2017) and **Guo et al.** (2019), who highlighted the importance of magnet arrangement strategies to suppress cogging and vibration in precision electromechanical systems [28, 40]. These reductions are particularly valuable for electric vehicle drives and positioning applications requiring high accuracy and minimal vibration.

Electromagnetic losses—specifically eddy current and hysteresis losses—were minimized through optimized lamination and integrated cooling systems, consistent with recommendations by **Hosoya et al.** (2011) and **Zhang et al.** (2023) [29, 31]. Thermal simulations further revealed stable temperature profiles across the magnetic cores, crucial for preserving magnet remanence and preventing demagnetization under high-flux conditions.

The high heat flux near the Halbach array's flux concentration zones demanded robust thermal management. Finite element thermal analysis confirmed that integrating micro-channel cooling and thermal interface materials could sustain safe operating temperatures, supporting previous research by **Li et al.** (2015) and **Wang et al.** (2018), emphasizing the importance of thermal-mechanical co-design in high-density magnetic systems [37, 39]. Similarly, mechanical stress analyses verified that reinforced structures withstand Lorentz forces effectively during high-torque operations, consistent with **Chau et al.** (2010) and **Zhao & Ji** (2017) [42, 49].

The mechanical strength and thermal stability are interdependent, where advancements in one domain often drive improvements in the other. Future investigations could explore the integration of composite materials, micro-channel liquid cooling, and additive-manufactured components to further enhance performance.

Large-scale optimization employing advanced algorithms such as genetic algorithms (GA) and particle swarm optimization (PSO) enabled systematic refinement of magnet configuration and winding geometry. This approach aligns with **Li, Qu, & Lipo** (2014) and **Zhu & Howe** (2007), who established that intelligent optimization significantly enhances efficiency while lowering operational costs [34, 36]. The sensitivity and robustness analyses confirmed that the proposed HAPMVM maintains consistent performance despite manufacturing tolerances and external disturbances.

However, practical implementation of the HAPMVM design requires consideration of manufacturing constraints, including magnet placement precision, assembly alignment, and quality assurance. As emphasized by **Lin & Chau** (2018) and **Li & Qu** (2012), maintaining strict fabrication tolerances is essential to realizing the theoretical performance gains predicted by simulations [41, 47].

The proposed HAPMVM exhibits high torque density, minimal torque ripple, and efficient flux utilization, making it a strong candidate for renewable applications such as wind and tidal power generation, as well as electric mobility systems. Its scalable and modular structure supports multi-power configurations, while its low electromagnetic interference enables integration into sensitive control systems. Thus, embedding Halbach arrays in Vernier machines represents a major advancement toward compact, efficient, and reliable electric drives aligned with the global shift to sustainable energy technologies [45, 50].

6. Conclusion

The integration of Halbach arrays in Vernier machine topologies has shown a great opportunity in improving the performance of electromagnetism, torque density, and efficiency of operations. Simulation, systematic design and optimization strategy that is followed in the current study is a basis of experimental validation and industrial implementation in the future. With electric machines becoming a growing part of the renewable energy and transportation industries, alternative layouts such as the suggested HAPMVM will become essential towards facilitating sustainable and efficient power-to-energy methods.

7. Limitations and Future Directions

Although the simulation outcomes are encouraging, experimental validation is also necessary in order to verify the feasibility of the design. The intricate flux traces and the focused flux areas demand the careful production and assembly methods. Also, the design can become even more optimized by considering alternative magnetic materials with greater thermal stability and reduced cost. The use of machine learning algorithms to control in real-time and optimize adaptively as proposed by Li et al. [2014] could also be included in future research. These developments will enable the implementation of high performing electric machines in various working conditions.

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