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AN INNOVATIVE APPROACH OF ENGINEERING OPTIMIZATION

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Abstract
Optimization methods have various applications in the field of science and engineering. The construction, design and maintenance of engineering systems involve decision making both at the technological and the managerial level. In today's era, engineers strive to achieve optimal solutions with high accuracy and minimal computational time, leveraging the power of emerging artificial intelligence (AI) technologies. The effectiveness of traditional optimization methods can be enhanced by machine learning model trainers. Among the various optimization methods, the quasi-Newton methods are developed and proved to overcome the limitations of other classical methods. Multi-step quasi-Newton methods are the best version of QN method which approximate the current Hessian by using the data from more than one previous step. Several nonlinear problems were solved by using multi-step quasi-Newton method with skipping and modified search direction technique on variable space. In this study, 3-step quasi-Newton methods with different techniques of skipping and modified search direction on gradient space is experimented. On the basis of numerical results, the behavior of proposed strategy is compiled and found the superiority of fixed-point approach.

INTRODUCTION

Optimization plays key role in the field of science, economics, engineering etc. For instance, in engineering the optimal design of structural units in construction, machinery and in space vehicles, optimal path finding in road networks, planning for best production, controlling, and scheduling and optimal allocation of resources or best services among several activities is needed. Therefore, all the situations are represented by a function which requires to optimize i.e., to maximize the benefit and minimize the product costs in various manufacturing and construction processes with or without constraints [1]. Mathematically express as:

$$Optimize\ f(x) \tag{1}$$

Subjected to

$$a_i(\boldsymbol{X}) = 0$$
$$b_i(\boldsymbol{X}) \leq 0, \tag{2}$$

$$c_i(\boldsymbol{X}) \geq 0,$$

where $X = (x1, x2, \ldots\ldots xn)$ is n-dimensional vector, $f(X)$ is an objective function and Eq. (2) is set of constraints. In recent years, several well-defined methods including classical and meta-heuristic methods in optimization theory are developed and implemented for the solution of constrained/unconstrained and linear/ nonlinear optimization problems. The comparative analysis of these methods helps the end-users to select best method for the situation. Nagham [2] analyzed that how artificial intelligence is revolutionizing engineering by transforming labor-intensive and time- consuming tasks into processes defined by precision, speed, and optimized cognitive effort. Quasi-Newton methods (QN) [3 - 7] was proved as the reliable method for unconstrained problems. Berahas et al. [8] employed two sampled quasi-Newton methodsŮsampled LBFGS and sampled LSR1Ů are to address empirical risk minimization problems commonly encountered in machine learning. It was further investigated as multi-step quasi-Newton methods (mQN) [9] and found remarkable achievements. Further developments [10, 11] with skipping and modified search direction techniques in mQN were successfully attained. The main idea of QN methods is to compute approximation of Hessian B_t or inverse Hessian B_t^{-1} which must satisfy the secant equation [12] defined as:

$$B_{t+1}s_t = y_t \tag{3}$$

Where s_t and y_t are the variable and gradient variations respectfully and the update of Hessian approximation matrix B_{t+1} is defined by one of the effective QN methods i.e. Broyden-Fletcher-Goldfarb-Shanno (BFGS) method as:

$$B_{t+1}^{BF\,GS} = B_t + \frac{y_t y_t^T}{y_t^T s_t} - \frac{B_t s_t s_t^T B_t}{s_t^T B_t s_t} \tag{4}$$

which guarantee the positive definiteness of B_{t+1} as well as B_t. It is evident from secant equation that approximate Hessian is updated with the information of only one previous step, so it is single step method. In mQN, the information of more than one step is needed for updating. Therefore, in case of 3-step method the secant equation takes the form as

$$B_{t+1}(s_t - \alpha_t s_{t-1} + \beta_t s_{t-2})$$

$$= y_t - \alpha_t y_{t-1} + \beta_t y_{t-2} \tag{5}$$

or

$$\boldsymbol{B_{t+1} r_t} = \boldsymbol{w_t} \tag{6}$$

The derivation detail of r_t and w_t can be found in [13]. Hence, BFGS formula for mQN is defined as:

$$B_{t+1} = B_t + \frac{(B_t r_t r_t^T B_t)}{r_t^T B_t r_t} + \frac{w_t w_t^T}{r_t^T w_t} \tag{7}$$

In Eq (5) the values of α_t and β_t are given as:

$$\alpha_t = \frac{-\delta_1^2 (\delta_2 + 1)^3}{(\delta_1 - \delta_2)(3\delta_1\delta_2 + \delta_1 + \delta_2)} + 1 \tag{8}$$

$$\beta_t = \frac{(\delta_1\delta_2)^2}{3\delta_{1\delta_2} + \delta_1 + \delta_2} \tag{9}$$

where,

$$\delta 1 = \frac{\tau_3 - \tau_1}{\tau_1 - \tau_0} \tag{10}$$

$$\delta_2 = \frac{\tau_3 - \tau_2}{\tau_2 - \tau_0} \tag{11}$$

In Eqs (10) and (11), $\{\tau_i\}_{i=0}^3$ are the parametric values computed by metric based method described by Ford and Moghrabi [9] by defining the metric as:

$$\varphi_S = \left((z_1 - z_2)^T S (z_1 - z_2)\right)^{\frac{1}{2}} \tag{12}$$

where S is $n \times n$ symmetric positive-definite matrix and $z_1, z_2 \in R^n$. In the literature [13, 16] the parameter $\tau's$ are calculated on variable space by two approaches of metric based method. In this study, we have successfully attained the experimental results of 3-step QN methods by the computation of parameters on gradient space

Proposed Strategy

Two approaches i.e. accumulative and fixed-point of metric based method are adopted for the computation of parametric values $\tau's$ on gradient space. The Eq. (12) is used to calculate the metric between the most recent gradient space $\{gt - m + n + 1\}_{n=0}^m$. Therefore, In accumulative approach the iterates are calculated on their natural pattern defined as:

$$\tau_n = \tau_{n+1} - \varphi_S(g_{t-m+n+2}, g_{t-m+n+1}),$$
$$\text{for } n = 0, 1, \ldots. m - 1 \tag{13}$$

and in fixed-point approach iterates are computed by keeping one iterate fixed given as:

$$\tau_n = -\varphi_S(g_{t+1}, g_{t+k-n+1}),$$
$$\text{for } n = 0, 1, \ldots., m \tag{14}$$

We have taken three matrices for matrix S i.e.
$S = I$ (Identity)
$S = H_t$ (Inverse Hessian)
and $S = H_{t+1}$ (Updated inverse Hessian)
The selected matrices used in the approaches of metric based method for the computation of parameters $\tau's$ for 3-step quasi-Newton methods.
The general algorithm of proposed method is as follow:

- Select accumulative/fixed point approach.
- Consider $\tau_3 = 0$ as base point.
- Select matrix S as I, Hi and $Hi + 1$
- Assign $n = 2$ for three-step methods
- Calculate $\tau 0s$, by accumulative approach $\tau_n = \tau_{n+1} - \varphi_S(g_{t-m+n+2}, g_{t-m+n+1})$
- Compute $\tau_n = -\varphi_S(g_{t+1}, g_{t+k-n+1})$ by fixed-point approach.
- Decrease n by 1, and goto step 5.

Table 1: Abbreviations for Accumulative and Fixed-point Approach

Approach	Abbreviation in 3-step
Accumulative (AS)	A_I A_{H_t} $A_{H_{t+1}}$
Fixed Point (FS)	F_I F_{Ht} $F_{H_{t+1}}$

2.1 Accumulative Approach

Taking $\tau_m = 0$, other τ_s' are calculated by Eq. (13).

Accumulative Approach in Three-step Method

In three-step method, we have taken $\tau_3 = 0$ as an origin. The other parameters τ_0, τ_1 and τ_2 are found by Eq. (13) by putting $k = 0,1,2$ respectively.

- **Algorithm A_I**

The choice of matrix $\boldsymbol{S}$ in this algorithm is Identity matrix $(\boldsymbol{I})$ and parametric values are computed as:

$$\tau_2 = \tau_3 - \phi_I(g_{t-3+2+2}, g_{t-3+2+1}) \quad (15)$$
$$= -\phi_I(g_{t+1}, g_t) \quad (16)$$
$$= -\sqrt{y_t^T y_t} \quad (17)$$
$$\tau_1 = \tau_2 - \phi_I(g_{t-3+1+2}, g_{t-3+1+1}) \quad (18)$$
$$= -\sqrt{y_t} + \sqrt{y_{t-1}} \quad (19)$$
$$\tau_0 = \tau_1 - \phi_I(g_{t-3+0+2}, g_{t-3+0+1}) \quad (20)$$
$$= -\sqrt{y_t} + \sqrt{y_{t-1}} - \phi_I(g_{t-1}, g_{t-2}) \quad (21)$$

since we have $y_t = g_{t+1} - g_t$ if we replace t by $t-2$, we obtain $y_{t-2} = g_{t-1} - g_{t-2}$. Therefore the Eq. (21) will be as follow:

$$\tau_0 = -\sqrt{y_t} + \sqrt{y_{t-1}} + \sqrt{y_{t-2}} \quad (22)$$

- **Algorithm A_{H_t}**

In this algorithm, the matrix S is taken to be inverse Hessian approximation H_t and τ_s' are calculated by Eq. (13) as:

$$\tau_2 = \tau_3 - \phi_{H_t}(g_{t-3+2+2}, g_{t-3+2+1}) \quad (23)$$
$$= -\sqrt{y_t^T s_t} \quad (24)$$
$$\tau_1 = \tau_2 - \phi_{H_t}(g_{t-3+1+2}, g_{t-3+1+1}) \quad (25)$$
$$= -\sqrt{y_t^T s_t} - \sqrt{y_{t-1}^T s_{t-1}} \quad (26)$$
$$\tau_0 = \tau_1 - \phi_{H_t}(g_{t-3+0+2}, g_{t-3+0+1}) \quad (27)$$
$$\Rightarrow \tau_0 \approx -\sqrt{y_t^T s_t} - \sqrt{y_{t-1}^T s_{t-1}} - \sqrt{y_{t-2}^T s_{t-2}} \quad (28)$$

- **Algorithm $A_{H_{t+1}}$**

In this case choice for $\boldsymbol{S}$ is H_{t+1} (updated inverse Hessian) and for taking n=0,1,2 the Eq. (13) is used to compute the parametric values:

$$\tau_2 = \tau_3 - \phi_{H_{t+1}}(g_{t-3+2+2}, g_{t-3+2+1}) \quad (29)$$
$$\tau_2 = -\phi_{H_{t+1}}(g_{t+1}, g_t) \quad (30)$$
$$= -\sqrt{y_t^T s_t} \quad (31)$$
$$\tau_1 = \tau_2 - \phi_{H_{t+1}}(g_{t-3+1+2}, g_{t-3+1+1}) \quad (32)$$
$$= -\sqrt{y_t^T s_t} + \sqrt{y_{t-1}^T s_{t-1}} \quad (33)$$

$$\tau_0 = \tau_1 - \phi_{H_{t+1}}(g_{t-3+0+2}, g_{t-3+0+1}) \quad (34)$$

$$= -\sqrt{y_t^T s_t} + \sqrt{y_{t-1}^T s_{t-1}} - \phi_{H_{t+1}}(g_{t-1}, g_{t-2}) \quad (35)$$

with the help of $H_{t+1} y_{t-2} \approx s_{t-2}$, we get:

$$\tau_0 \approx -\sqrt{y_t^T s_t} + \sqrt{y_{t-1}^T s_{t-1}} + \sqrt{y_{t-2}^T s_{t-2}} \quad (36)$$

2.2 The Fixed-point Methods

The metric φ_S is used to find the distance of each iterate from a fixed point. The parametric values τ_n are computed with the help of Eq. (14) while taking $\tau_m = 0$.

Fixed-point Approach in Three-step Method

In three-step methods we have $\tau_3 = 0$ and other parametric values are computed by Eq. (14) with the selected matrices for the matrix $\boldsymbol{S}$.

- **Algorithm F_I**

In this case, the choice of matrix N is identity matrix and by using Eq. (14) we have computed parametric values as:

$$\tau_2 = -\phi_I(g_{t+1}, g_{t+2-3+1}) \quad (37)$$

$$= -\phi_I(g_{\{t+1\}}, g_t) = -\|y_t\|_2 \quad (38)$$

$$\tau_1 = -\phi_I(g_{t+1}, g_{t+1-3+1}) \quad (39)$$

$$= -\phi_I(g_{t+1}, g_{t-1}) \quad (40)$$

$$= -\sqrt{y_t} + \sqrt{y_{t-1}} \quad (41)$$

$$\tau_0 = -\phi_I(g_{t+1}, g_{t+0-3+1}) \quad (42)$$

$$= -\phi_I(g_{t+1}, g_{t-2}) \quad (43)$$

$$= -\sqrt{y_t} + \sqrt{y_{t-1}} + \sqrt{y_{t-2}} \quad (44)$$

- **Algorithm F_{H_t}**

In this algorithm, matrix $\boldsymbol{S}$ is selected to be H_t in Eq. (14) for the computation of parametric values, we get:

$$\tau_2 = -\phi_{H_t}(g_{t+1}, g_{t+2-3+1}) \quad (45)$$

$$= -\phi_{H_t}(g_{t+1}, g_t) \quad (46)$$

$$= -\sqrt{y_t^T s_t} \quad (47)$$

$$\tau_1 = -\phi_{H_t}(g_{t+1}, g_{t-1}) \quad (48)$$

$$= -\sqrt{-y_t^T s_t - 2y_{t-1}^T s_t + y_{t-1}^T s_{t-1}} \quad (49)$$

$$\tau_0 = -\phi_{H_t}(g_{t+1}, g_{t-2}) \quad (50)$$

$$= -\sqrt{y_t^T s_t + 2y_{t-1}^T s_t + 2y_{t-2}^T s_t + 2y_{t-1}^T s_{t-1} + y_{t-2}^T s_{t-2}} \quad (51)$$

- **Algorithm $F_{H_{t+1}}$**

In this algorithm, τ_2, τ_1 and τ_0 are computed by taking $\boldsymbol{S} = \boldsymbol{H_{t+1}}$. Therefore from Eq. (14) we obtain parametric values as:

$$\tau_2 = -\phi_{H_{t+1}}(g_{t+1}, g_{t+2-3+1}) \quad (52)$$

$$= -\sqrt{y_t^T s_t} \quad (53)$$

$$\tau_1 = -\phi_{H_{t+1}}(g_{t+1}, g_{t+1-3+1}) \quad (54)$$

$$= -\sqrt{y_t^T s_t + 2y_{t-1}^T s_t + y_{t-1}^T s_{t-1}} \quad (55)$$

$$\tau_0 = -\phi_{H_{t+1}}(g_{t+1}, g_{t-2}) \quad (56)$$

$$= -\sqrt{y_t^T s_t + 2y_{t-1}^T s_t + 2y_{t-2}^T s_t + 2y_{t-2}^T s_{t-2} + y_{t-1}^T s_{t-1} + y_{t-2}^T s_{t-2}} \quad (57)$$

3 Numerical Results and Discussion

Different test functions from [15] are taken in three categories of soft, medium, and hard dimension ranging from 2 to 200. Soft problems are ranging from 2 to 20, medium dimension ranging from 21 to 60 and hard problems have range from 61 to 200, mentioned in Table 2. Four different starting points and epsilon values for each test function are reported in Table 2). These test functions are performed by single-step BFGS, unit-spaced and multi-step methods with different techniques. Experimental results of three-step quasi-Newton methods with different techniques on gradient space are compiled under certain notations.

3.1 Notations

The notation of method is represented as ${}^{[2]}_{[2]}Method_n^{(3)}$. This notation denotes the 3-step QN methods with 2 updates skipped with modified search direction. The following table also specifies the metric-based methods used in 3-step QN methods with three choices of symmetric matrix $\boldsymbol{S}$. In 3-step methods the techniques of skipping and modified search direction (see detail in [13])

Notation	Discription
${}^{[2]}_{[2]}\boldsymbol{A}^{(3)}_{\boldsymbol{n}}$	Accumulative three-step method with two updates skipped and modified search direction
${}^{[2]}_{[2]}\boldsymbol{F}^{(3)}_{\boldsymbol{n}}$	Fixed point three-step method with two updates skipped and modified search direction

Note: $n = 1,2,3$ corresponds to matrices I, H_t, H_{t+1} respectively.

are employed on gradient space. We have checked and analyzed the performance of all the methods on the test problems which are taken from [15] in soft, medium, and hard dimension listed in Table 3. The parametric values on gradient space of three step methods are also calculated. The results of different methods such as accumulative and fixed-point approaches with three choices of matrix $\boldsymbol{S}$ are obtained and compared. The conclusion is drawn after the comparison of these methods. The experimental results are displayed in Table 2.

- It can be observed from Table 2, that ${}^{[2]}_{[2]}F^{(3)}_3$ outperformed in function evaluation and gave the results in minimum. On the other hand, ${}^{[2]}_{[2]}A^{(3)}_1$ computed the results in less
- number of iteration in soft dimension problems.
- In medium dimension, ${}^{[2]}_{[2]}F^{(3)}_3$ best result in function evaluation, ${}^{[2]}_{[2]}A^{(3)}_1$ method exhibits less number of iterations and ${}^{[2]}_{[2]}F^{(3)}_1$ performed execution in less time duration.
- The results of hard dimension problems showed that the method ${}^{[2]}_{[2]}F^{(3)}_3$ produced best results in term of function evaluation and number of iteration while ${}^{[2]}_{[2]}F^{(3)}_1$ evaluated the results in minimum time.
- In combined, It is obvious that the method ${}^{[2]}_{[2]}F^{(3)}_3$, ${}^{[2]}_{[2]}A^{(3)}_1$ and ${}^{[2]}_{[2]}A^{(3)}_2 1$ outperformed in function evaluation, number of iterations and time in seconds respectively.

Table 2: Result of three-step methods of all Dimension

Method	**F. Evaluation**	**Iteration**	**Time (sec)**	**Dimension**
${}^{[2]}_{[2]}A^{(3)}_1$	17463	10721	4.3482	Soft
${}^{[2]}_{[2]}F^{(3)}_1$	23693	11530	4.8524	Soft
${}^{[2]}_{[2]}A^{(3)}_2$	19638	12699	5.2990	Soft
${}^{[2]}_{[2]}F^{(3)}_2$	17951	12389	5.2665	Soft
${}^{[2]}_{[2]}A^{(3)}_3$	19638	12699	4.6191	Soft
${}^{[2]}_{[2]}F^{(3)}_3$	16278	11650	4.3423	Soft
${}^{[2]}_{[2]}A^{(3)}_1$	16959	14226	6.0819	Medium
${}^{[2]}_{[2]}F^{(3)}_1$	19268	14619	5.6075	Medium
${}^{[2]}_{[2]}A^{(3)}_2$	20705	17790	6.8455	Medium
${}^{[2]}_{[2]}F^{(3)}_2$	18483	15903	6.8391	Medium
${}^{[2]}_{[2]}A^{(3)}_3$	20705	17790	6.6854	Medium

${}^{[2]}_{[2]}F_3^{(3)}$	16483	14234	6.0652	Medium
${}^{[2]}_{[2]}A_1^{(3)}$	39222	35921	35.7930	Hard
${}^{[2]}_{[2]}F_1^{(3)}$	41309	37814	31.2645	Hard
${}^{[2]}_{[2]}A_2^{(3)}$	52749	48807	47.6138	Hard
${}^{[2]}_{[2]}F_2^{(3)}$	43723	40094	33.9750	Hard
${}^{[2]}_{[2]}A_3^{(3)}$	52749	48807	40.1080	Hard
${}^{[2]}_{[2]}F_3^{(3)}$	38628	35660	31.6481	Hard
${}^{[2]}_{[2]}A_1^{(3)}$	73644	60868	46.2230	Combined
${}^{[2]}_{[2]}F_1^{(3)}$	84270	63963	41.7245	Combined
${}^{[2]}_{[2]}A_2^{(3)}$	93092	79296	59.7584	Combined
${}^{[2]}_{[2]}F_2^{(3)}$	80157	68386	46.0807	Combined
${}^{[2]}_{[2]}A_3^{(3)}$	93092	79296	51.4125	Combined
${}^{[2]}_{[2]}F_3^{(3)}$	71389	61544	42.0555	Combined

Conclusion

Different dimension problems are tested by 3-step QN methods with the techniques of skipping and modified search direction. To check the performance of 3-step methods the parametric values τ_s' are calculated on gradient space rather than variable space. The numerical results revealed that fixed-point approach showed significance performance in function evaluation, and it executed the test problems in minimum time. It is also found that minimum iterations are reported in all dimension problems by accumulative approach.

Table 3: Test Problems and Dimensions [16]

Function Name	**Dimensions**
Extended Rosenbrock	2, 20, 26, 40, 60, 80, 100, 120
Extended Wood	4, 12, 24, 48, 68, 92, 112, 140
Extended Powell Singular	4, 8, 60, 80, 100, 140
Penalty 1	10, 14, 20, 30
Penalty 2	10, 16, 24, 30
Modified Trigonometric Function	16, 32, 64, 95, 128, 150
Broyden Tridiagonal	18, 36, 72, 90, 108, 144, 186
Discrete Boundary Value	20, 38, 60, 90, 120, 136, 188

Discrete Integral Equation	20, 84, 100, 150, 175, 200
Freudenstein and Roth	28, 52, 85, 118, 190
Variably Dimensioned	30, 55, 75, 100, 130, 150
Merged Quadratic	**30, 50, 70, 110, 136, 180**
Discrete ODE II	33, 44, 66, 88, 110, 176
Discrete ODE I	42, 58, 78, 96, 114, 160
Extended Engvall Function	64, 76, 88, 104, 155, 196

In future, we are planning to expand our approach to four or five-step methods with different techniques. Different engineering problems can also be considered as test problems for optimal solution.

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